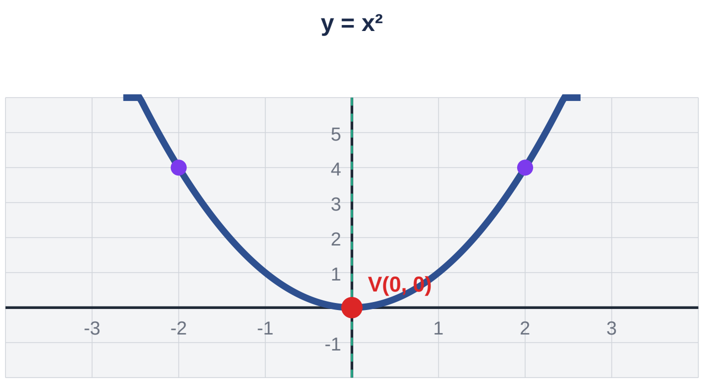
QUADRATICS MASTERY UNIT

Factoring to the Quadratic Formula

Grades 9 - 10 | Visual Guided Notes



Week 1	Week 2	Week 3	Week 4
GCF & Trinomial Factoring	Solving by Factoring & Completing the Square	Quadratic Formula & The Discriminant	Graphing, Transformations & Applications

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UNIT OVERVIEW & LEARNING TARGETS

This 4-week mastery unit covers quadratic functions from factoring foundations through all solution methods and graphing in three forms. By the end of this unit, you will be able to:

1. Factor expressions using the Greatest Common Factor (GCF).

2. Factor trinomials of the form $x^2 + bx + c$ and $ax^2 + bx + c$.

3. Recognize and factor the difference of squares and factor by grouping.

4. Solve quadratic equations by factoring (zero product property).

5. Solve quadratic equations by completing the square.

6. Apply the quadratic formula and interpret the discriminant.

7. Classify solutions as two real, one real (double), or no real solutions.

8. Graph parabolas in standard form, vertex form, and factored form.

9. Identify transformations of the parent function $y = x^2$.

10. Model and solve real-world projectile motion problems.

Key Vocabulary

Quadratic	**Parabola**	**Vertex**	**Axis of Symmetry**
Standard Form	**Vertex Form**	**Factored Form**	**Discriminant**
GCF	**Zero Product Property**	**Completing the Square**	**Projectile Motion**

LESSON 1: Greatest Common Factor (GCF)

Key Concept: What is Factoring?

Factoring is the **reverse** of the distributive property. Instead of multiplying out, we "un-multiply" to write an expression as a product of its factors.

Distributing: $3(x + 4) = 3x + 12$

Factoring: $3x + 12 = 3(x + 4)$

How to Find the GCF

Step 1 Find the GCF of the **coefficients** (numbers).

Step 2 Find the GCF of the **variables** (take the smallest exponent).

Step 3 Divide each term by the GCF and write the result in parentheses.

Example 1: Factor $6x^2 + 9x$

Step 1: GCF of 6 and 9 is **3**

Step 2: GCF of x^2 and x is $\mathbf{x}$

Step 3: Combined GCF = $\mathbf{3x}$

$6x^2 \div 3x = 2x$ and $9x \div 3x = 3$

$$6x^2 + 9x = 3x(2x + 3)$$

Example 2: Factor $12a^3b - 8a^2b^2 + 4ab$

Coefficients: GCF of 12, 8, 4 is **4**

Variables: GCF of a^3, a^2, a is $\mathbf{a}$; GCF of b, b^2, b is $\mathbf{b}$

Combined GCF = 4ab

$12a^3b \div 4ab = 3a^2$

$8a^2b^2 \div 4ab = 2ab$

$4ab \div 4ab = 1$

$$12a^3b - 8a^2b^2 + 4ab = 4ab(3a^2 - 2ab + 1)$$

LESSON 1 (continued): GCF Practice

Tip

Always check your answer by distributing the GCF back. If you get the original expression, your factoring is correct!

Example 3: Factor $-5x^3 + 10x^2 - 15x$

GCF of coefficients: 5 (factor out the negative too!)

GCF of variables: x

Combined GCF = $-5x$

$-5x^3 \div (-5x) = x^2$

$10x^2 \div (-5x) = -2x$

$-15x \div (-5x) = 3$

$$-5x^3 + 10x^2 - 15x = -5x(x^2 - 2x + 3)$$

Your Turn: Factor each expression

1. $4x + 8$

2. $6y^2 - 3y$

3. $10a^2b + 15ab$

4. $12m^3 - 8m^2 + 4m$

5. $-7p^2q + 14pq^2$

6. $9r^4 + 18r^3 - 27r^2$

7. $16x^3y - 24x^2y^2 + 8xy^3$

8. $-3n^2 + 6n - 12$

9. $20c^5d^2 + 15c^4d - 10c^3$

10. $2k(k + 3) + 5(k + 3)$

LESSON 2: Factoring Trinomials (a = 1)

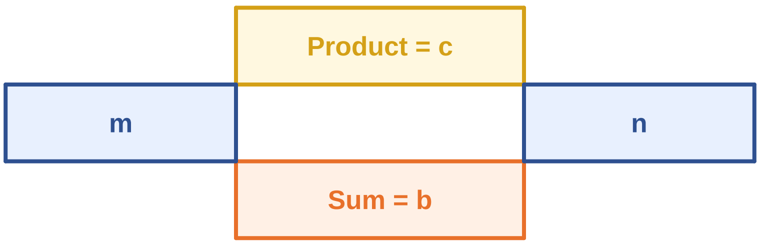
Key Concept: Factoring $x^2 + bx + c$

When the leading coefficient is 1, we look for two numbers that **multiply to c** and **add to b**.

$$x^2 + bx + c = (x + m)(x + n) \text{ where } m \cdot n = c \text{ and } m + n = b$$

The Diamond Method

The diamond diagram helps you organize your thinking. Place the **product** (c) on top and the **sum** (b) on the bottom. Find two numbers for the sides that make both true.



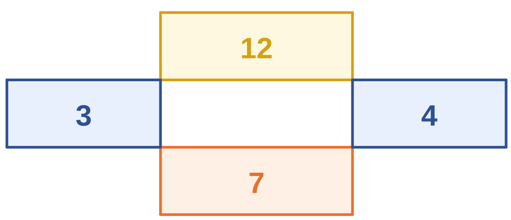
Example 1: Factor $x^2 + 7x + 12$

We need $m + n = \mathbf{7}$ and $m \cdot n = \mathbf{12}$

Pairs that multiply to 12: (1,12), (2,6), (3,4)

Only (3, 4) adds to 7. ✓

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$



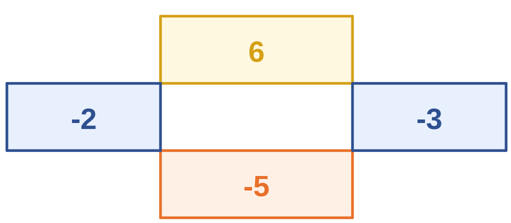
Example 2: Factor $x^2 – 5x + 6$

We need $m + n = –\mathbf{5}$ and $m \cdot n = \mathbf{6}$

Both factors must be negative (pos. product, neg. sum).

$(–2) + (–3) = –5$ and $(–2)(–3) = 6$ ✓

$$x^2 – 5x + 6 = (x – 2)(x – 3)$$



LESSON 2 (continued): More Trinomials & Practice

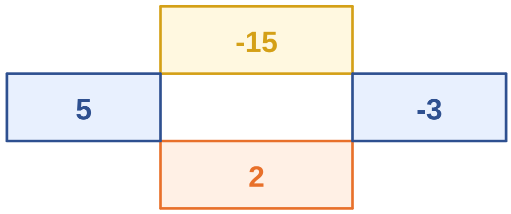
Example 3: Factor $x^2 + 2x - 15$

We need $m + n = \mathbf{2}$ and $m \cdot n = \mathbf{-15}$

One factor positive, one negative (neg. product).

$5 + (-3) = 2$ and $5(-3) = -15$ ✓

$$x^2 + 2x - 15 = (x + 5)(x - 3)$$



Sign Patterns for $x^2 + bx + c$

$c > 0, b > 0$: both factors positive → $(x + m)(x + n)$

$c > 0, b < 0$: both factors negative → $(x - m)(x - n)$

$c < 0$: one positive, one negative → $(x + m)(x - n)$

Tip

When c is negative, the larger absolute value takes the sign of b. For example, in $x^2 + 2x - 15$, the factor with larger |value| is +5 (same sign as b = +2).

Your Turn: Factor each trinomial

1. $x^2 + 5x + 6$

2. $x^2 + 9x + 14$

3. $x^2 - 4x + 3$

4. $x^2 - 8x + 15$

5. $x^2 + 3x - 10$

6. $x^2 - 2x - 24$

7. $x^2 + x - 20$

8. $x^2 - x - 12$

9. $x^2 + 11x + 28$

10. $x^2 - 9x + 20$

11. $x^2 + 4x - 21$

12. $x^2 - 7x - 18$

FULLY WORKED ANSWER KEY & SOLUTIONS

Unit Test: Fully Worked Solutions

Part 1: Factoring

1. $6x^2 - 12x$: The greatest common factor of 6 and 12 is 6, and both terms contain 'x'. Divide each term by 6x: $6x^2/6x = x$ and $-12x/6x = -2$. **$6x(x - 2)$**

2. $x^2 - 8x + 15$: We need two numbers that multiply to 15 and add to –8. The numbers –3 and –5 work because $(-3)(-5) = 15$ and $(-3) + (-5) = -8$. **$(x - 3)(x - 5)$**

3. $3x^2 - 14x + 8$: Use the AC method. Multiply a and c: $3 * 8 = 24$. Find factors of 24 that add to –14: –2 and –12. Split the middle term: $3x^2 - 2x - 12x + 8$. Factor by grouping: $x(3x - 2) - 4(3x - 2)$. **$(3x - 2)(x - 4)$**

4. $4x^2 - 25$: This is a difference of squares where $a^2 = 4x^2$ (so $a = 2x$) and $b^2 = 25$ (so $b = 5$). Apply the rule $a^2 - b^2 = (a - b)(a + b)$. **$(2x - 5)(2x + 5)$**

5. $x^3 - 3x^2 - 4x + 12$: Group the first two terms and the last two terms: $(x^3 - 3x^2) + (-4x + 12)$. Factor out the GCF from each group: $x^2(x - 3) - 4(x - 3)$. Factor out the common binomial $(x - 3)$. The remaining factor $(x^2 - 4)$ is also a difference of squares. **$(x - 3)(x - 2)(x + 2)$**

6. $2x^4 - 32$: First, factor out the GCF of 2: $2(x^4 - 16)$. The binomial inside is a difference of squares: $a^2 = x^4$ (so $a = x^2$) and $b^2 = 16$ (so $b = 4$). Factor to get $2(x^2 - 4)(x^2 + 4)$. Notice $(x^2 - 4)$ is also a difference of squares. **$2(x - 2)(x + 2)(x^2 + 4)$**

Part 2: Solving

7. Factor the quadratic: $(x - 4)(x - 5) = 0$. Set each factor to zero using the Zero Product Property: $x - 4 = 0$ or $x - 5 = 0$. **$x = 4, 5$**

8. $AC = -6$. Find factors of -6 that add to 5: $+6$ and -1. Split the middle: $2x^2 + 6x - x - 3 = 0$. Group: $2x(x + 3) - 1(x + 3) = 0$. Factor: $(2x - 1)(x + 3) = 0$. Set to zero: $2x - 1 = 0 \rightarrow x = 1/2$; $x + 3 = 0 \rightarrow x = -3$. **$x = 1/2, -3$**

9. Recognize the perfect square trinomial: $(x + 5)^2 = 0$. Take the square root: $x + 5 = 0$. **$x = -5$ (double root)**

10. CTS: Move the constant: $x^2 - 6x = -2$. Take half of -6, which is -3, and square it to get 9. Add 9 to both sides: $x^2 - 6x + 9 = 7$. Factor the perfect square: $(x - 3)^2 = 7$. Take the square root: $x - 3 = \pm\sqrt{7}$. Solve for x: **$x = 3 \pm \sqrt{7}$**

11. QF: Identify $a=3$, $b=-2$, $c=-4$. Substitute into the formula: $x = (2 \pm \sqrt{(-2)^2 - 4(3)(-4)}) / (2*3)$. Simplify the radicand: $4 + 48 = 52$. Result: $x = (2 \pm \sqrt{52})/6$. Simplify the radical ($\sqrt{52} = 2\sqrt{13}$) and reduce the fraction. **$x = (1 \pm \sqrt{13})/3$**

12. QF: $a=1$, $b=4$, $c=8$. Substitute: $x = (-4 \pm \sqrt{16 - 32})/2 = (-4 \pm \sqrt{-16})/2$. Replace $\sqrt{-16}$ with $4i$. Result: $(-4 \pm 4i)/2$. **$x = -2 \pm 2i$**

Lesson 6: Completing the Square Solutions

1. Move 5: $x^2 + 8x = -5$. Half of 8 is 4, squared is 16. Add 16: $(x + 4)^2 = 11$. **$x = -4 \pm \sqrt{11}$**

2. Move –7: $x^2 - 4x = 7$. Half of –4 is –2, squared is 4. Add 4: $(x - 2)^2 = 11$. **$x = 2 \pm \sqrt{11}$**

3. Move 30: $x^2 + 12x = -30$. Half of 12 is 6, squared is 36. Add 36: $(x + 6)^2 = 6$. **$x = -6 \pm \sqrt{6}$**

4. Move –5: $x^2 - 2x = 5$. Half of –2 is –1, squared is 1. Add 1: $(x - 1)^2 = 6$. **$x = 1 \pm \sqrt{6}$**

5. Divide by 2: $x^2 + 6x - 4 = 0$. Move –4: $x^2 + 6x = 4$. Half of 6 is 3, squared is 9. Add 9: $(x + 3)^2 = 13$. **$x = -3 \pm \sqrt{13}$**

6. Divide by 3: $x^2 - 2x + 1/3 = 0$. Move 1/3: $x^2 - 2x = -1/3$. Half of –2 is –1, squared is 1. Add 1: $(x - 1)^2 = 2/3$. **$x = 1 \pm \sqrt{6}/3$**

Lesson 7: Quadratic Formula Solutions

1. $a=1$, $b=-3$, $c=-10$. $x = (3 \pm \sqrt{(9 + 40)})/2 = (3 \pm 7)/2$. **$x = 5, -2$**

2. $a=1$, $b=6$, $c=9$. $x = (-6 \pm \sqrt{(36 - 36)})/2 = -6/2$. **$x = -3$ (double root)**

3. $a=2$, $b=3$, $c=-2$. $x = (-3 \pm \sqrt{(9 + 16)})/4 = (-3 \pm 5)/4$. **$x = 1/2, -2$**

4. $a=1$, $b=-2$, $c=2$. $x = (2 \pm \sqrt{(4 - 8)})/2 = (2 \pm 2i)/2$. **$x = 1 \pm i$**

5. $a=3$, $b=-2$, $c=-4$. $x = (2 \pm \sqrt{(4 + 48)})/6 = (2 \pm \sqrt{52})/6$. Simplify $\sqrt{52}$ to $2\sqrt{13}$. **$x = (1 \pm \sqrt{13})/3$**

6. $a=1$, $b=0$, $c=16$. $x = (0 \pm \sqrt{(-64)})/2 = \pm 8i/2$. **$x = \pm 4i$**

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