Pre-Calculus Visual Notes & Reference Pack

Cornell-Style Guided Notes

Functions | Trigonometry | Sequences | Conics | Limits
Worked Examples | Formula Cards | Practice | Answer Keys

UM Printables

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Quick-Reference Formula Card

Algebra & Functions

Formula Name	Formula
Slope Formula	$m = (y_2 - y_1) / (x_2 - x_1)$
Distance Formula	$d = \mathrm{sqrt}[(x_2 - x_1)^2 + (y_2 - y_1)^2]$
Midpoint Formula	$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$
Quadratic Formula	$x = [-b +/- \mathrm{sqrt}(b^2 - 4ac)] / 2a$
Difference Quotient	[f(x+h) - f(x)] / h
Vertical Line Test	A graph is a function if no vertical line intersects it more than once.

Logarithms & Exponentials

Property	Rule
Product Rule	$\log_a(M * N) = \log_a(M) + \log_a(N)$
Quotient Rule	$\log_a(M / N) = \log_a(M) - \log_a(N)$
Power Rule	$\log_a(M^p) = p * \log_a(M)$
Change of Base	$\log_a(M) = \log_b(M) / \log_b(a)$
Exponential Equation	$a^x = y$ <--> $x = \log_a(y)$

Unit 1: Functions and Graphs | Function Basics	
Cues / Questions	**Notes / Worked Examples**
What is a function?	A relation where every input (x) has exactly ONE output (y). **Not a function:** $x = y^2$ (fails vertical line test) **Function:** $y = x^2$
Domain vs. Range	**Domain:** Set of all possible x-values (inputs). **Range:** Set of all possible y-values (outputs).
Finding Domain (Denominators)	Denominators cannot equal zero. **Example:** f(x) = 1/(x - 3) Set denominator != 0: x - 3 != 0 --> x != 3 Domain: All real numbers except 3.
Finding Domain (Radicals)	Radicands of even roots must be >= 0. **Example:** f(x) = sqrt(4 - x) Set radicand >= 0: 4 - x >= 0 --> x <= 4 Domain: x <= 4 or (-infinity, 4].

Summary / Synthesis: *A function maps each input to exactly one output. The domain is restricted by division by zero (denominator != 0) and even roots of negative numbers (radicand >= 0). Always check for these restrictions first.*

Unit 1: Functions and Graphs | Transformations

Cues / Questions	Notes / Worked Examples
Parent Functions	Base graphs before transformations: - Linear: $y = x$ - Quadratic: $y = x^2$ - Absolute Value: $y = \|x\|$ - Square Root: y = sqrt(x)
Transformation Formula	y = a * f(b(x - h)) + k - **a:** Vertical stretch/shrink & reflection (if $a<0$) - **b:** Horizontal stretch/shrink & reflection (if $b<0$) - **h:** Horizontal shift (opposite sign) - **k:** Vertical shift
Worked Example: **$y = -2(x - 3)^2 + 4$**	1. Parent: $y = x^2$ 2. $a = -2$: Reflected over x-axis, vertical stretch by 2 3. $h = 3$: Shifted RIGHT 3 units 4. $k = 4$: Shifted UP 4 units

Summary / Synthesis: *Transformations alter the position or shape of a parent graph. Inside the function (x-h, bx) affects the x-values horizontally (opposite of what you think). Outside the function (af, +k) affects the y-values vertically (exactly as you think).*

Unit 1: Functions and Graphs | Operations & Compositions

Cues / Questions	Notes / Worked Examples
Function Operations	Given f(x) and g(x): - Addition: (f + g)(x) = f(x) + g(x) - Subtraction: (f - g)(x) = f(x) - g(x) - Multiplication: (f * g)(x) = f(x) * g(x) - Division: (f / g)(x) = f(x) / g(x), g(x) != 0
Function Composition	Plugging one function into another. (f o g)(x) = f(g(x)) *Work inside out!*
Worked Example: **$f(x) = 2x + 1$, $g(x) = x^2$**	**Find f(g(x)):** 1. Substitute g(x) into f(x): $f(x^2)$ 2. Apply f rule: $2(x^2) + 1 = 2x^2 + 1$ **Find g(f(x)):** 1. Substitute f(x) into g(x): g(2x + 1) 2. Apply g rule: $(2x + 1)^2 = 4x^2 + 4x + 1$
Inverse Functions **$f^{-1}(x)$**	Swaps x and y. Undoes the original function. 1. Replace f(x) with y. 2. Swap x and y. 3. Solve for y. *Verify: $f(f^{-1}(x)) = x$*

Summary / Synthesis: *Functions can be combined using arithmetic operations or composition (nesting). Composition (f o g) requires evaluating the inner function first. An inverse function reverses the mapping of the original, effectively swapping the domain and range.*

Unit 1 Practice: Functions and Graphs

1. Find the domain of f(x) = sqrt(x + 5) / (x - 2). Express your answer in interval notation.

2. Describe the transformation of g(x) = -|x + 4| - 1 from its parent function. What is the new vertex?

3. Given f(x) = 3x - 5 and g(x) = x^2 + 2, find (f o g)(x) and evaluate (f o g)(-1).

4. Find the inverse function, $f^{-1}(x)$, for f(x) = (2x + 1) / 3.

5. If f(x) = sqrt(x), explain why the domain must be restricted. What is the domain of f(x)?

Unit 2: Polynomial & Rational Functions | Polynomial Characteristics

Cues / Questions	Notes / Worked Examples
End Behavior Rules	**1. Even Degree, Positive Leading Coefficient:** Falls left, Rises right (Up/Up) **2. Even Degree, Negative Leading Coefficient:** Rises left, Falls right (Down/Down) **3. Odd Degree, Positive Leading Coefficient:** Falls left, Rises right (Down/Up) **4. Odd Degree, Negative Leading Coefficient:** Rises left, Falls right (Up/Down)
Multiplicity of Zeros	How a graph behaves at its x-intercepts: **Odd Multiplicity (1, 3, 5...):** Graph **CROSSES** the x-axis. **Even Multiplicity (2, 4, 6...):** Graph **BOUNCES OFF** (touches) the x-axis.
Worked Example: **$f(x) = -2x^3 + 8x$**	1. **Zeros:** Set $f(x) = 0$ --> $-2x(x^2 - 4) = 0$ $x = 0$ (mult 1), $x = 2$ (mult 1), $x = -2$ (mult 1) 2. **End Behavior:** Odd degree (3), negative lead coeff (-2). Rises left, Falls right. 3. **Sketch:** Crosses at -2, 0, and 2. Starts top-left, ends bottom-right.

Summary / Synthesis: *The end behavior of a polynomial is dictated solely by its degree and leading coefficient. The behavior at the x-intercepts depends on the multiplicity of the zeros: odd multiplicity crosses the axis, while even multiplicity bounces off.*

Unit 2: Polynomial & Rational Functions | Rational Functions

Cues / Questions	Notes / Worked Examples
Vertical Asymptotes vs. Holes	**Vertical Asymptote (VA):** Occurs at x = c if (x - c) is left in the denominator after simplifying. (Denominator = 0, Numerator != 0). **Hole (Removable Discontinuity):** Occurs at x = c if (x - c) cancels from both the numerator and denominator. *The graph is undefined there, but doesn't shoot to infinity.*
Horizontal Asymptotes (HA)	Compare the degree of the numerator (N) to the denominator (D): - **N < D:** HA is y = 0 - **N = D:** HA is y = (Lead Coeff of N) / (Lead Coeff of D) - **N > D:** No HA (There may be a Slant/Oblique Asymptote)
Worked Example: $f(x) = (x^2 - 4) / (x^2 + x - 6)$	1. **Factor:** [(x-2)(x+2)] / [(x+3)(x-2)] 2. **Hole:** (x-2) cancels --> Hole at x = 2. To find y-coordinate of hole, plug x=2 into simplified: (2+2)/(2+3) = 4/5. Hole at **(2, 4/5)**. 3. **VA:** Denominator of simplified: x + 3 = 0 --> **VA at x = -3**. 4. **HA:** N degree = D degree (2). y = 1/1 = 1 --> **HA at y = 1**.

Summary / Synthesis: *Rational functions are ratios of polynomials. Always factor and simplify first to identify holes. The remaining denominator zeros create vertical asymptotes. Horizontal asymptotes are found by comparing the highest degrees of the numerator and denominator.*

Unit 2 Practice: Polynomial & Rational Functions

1. Determine the end behavior and the zeros (with their multiplicities) for $f(x) = -2x^3 + 8x$.

2. Find the vertical asymptotes, horizontal asymptote, and any holes for $f(x) = (x^2 - 9) / (x^2 + x - 12)$.

3. Create a sign map/number line for $f(x) = (x^2 - 9) / (x^2 + x - 12)$ and state the intervals where the function is positive.

4. Explain the difference between a hole and a vertical asymptote in terms of the algebraic factors of a rational function.