MATH WORD PROBLEM MASTERY TOOLKIT

Multi-Step Problem Solving
Grades 6–9

130 Worked-Solution Problems

- 5-Step RRPSC Framework (Read, Represent, Plan, Solve, Check)
- 10 Visual Problem-Solving Strategies with Reference Cards
- 5 Domain Practice Sets — 20 Problems Each with Full Solutions
- 10 Worked-Example Analysis Tasks
- 30-Problem Mixed Multi-Step Challenge Set
- Student Self-Monitoring Checklist
- Differentiated Support Scaffolds for Struggling Learners
- Teacher Facilitation Guide with Extensions
- 20-Problem Unit Assessment with Complete Solutions Key

UM Printables

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The 5-Step RRPSC Framework

Every problem in this toolkit is solved using the **RRPSC Framework** — a structured, research-backed approach that transforms word problems from confusing blocks of text into clear, solvable tasks. Each step builds mathematical reasoning and prevents common errors.

R	**Read & Identify:** What is the question? What information is given? What domain?
R	**Represent:** Draw a visual model — bar model, table, equation, diagram, or graph.
P	**Plan:** Choose a strategy. Write the mathematical model or equation.
S	**Solve:** Execute the plan. Show all working clearly, step by step.
C	**Check:** Does the answer make sense? Verify with estimation or substitution.

R — Read & Identify

- Read the problem carefully at least twice.
- Identify the question: What exactly is being asked?
- Identify key information: What numbers, quantities, and relationships are given?
- Identify the mathematical domain: Is this about ratios, percentages, equations, geometry, or data?
- Underline or highlight important words and numbers.
- Watch for hidden information (e.g., "together" implies addition).

R — Represent

- Choose a visual model that matches the problem structure:
- • Bar/Tape Model — best for part-whole and comparison problems
- • Table or Organised List — best for patterns and multiple cases
- • Equation or Inequality — best for relationships between variables
- • Diagram — best for spatial and geometry problems
- • Graph — best for relationships that change or compare
- The visual model makes the abstract concrete and reveals the solution path.

P — Plan

- Select one of the 10 problem-solving strategies (see Strategy Reference Cards).
- Write the mathematical model: an equation, proportion, or expression.
- Identify the operations needed and the order of calculations.
- For multi-step problems, break the solution into sequential sub-goals.
- Write your plan in words before computing: "First I will… Then I will…"

S — Solve

- Carry out each calculation carefully, showing all working.
- Write neatly and align your steps.
- Check each arithmetic step before moving on.
- Include units in intermediate and final answers where appropriate.

- If stuck, revisit your Plan (P) or Representation (R).

C — Check

- Ask: "Does my answer make sense in the context of the problem?"
- Use estimation to confirm reasonableness.
- Substitute the answer back into the original equation or condition.
- Check for common errors: wrong operation, misplaced decimal, missing unit.
- Write a complete sentence answering the original question.

Visual Strategy Reference Cards

These 10 strategies are your toolkit for solving multi-step word problems. Keep these cards in your binder as a quick reference for when you get stuck on the **Plan** step.

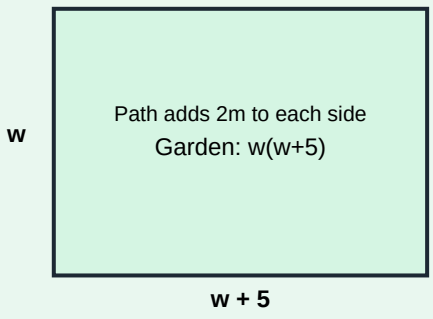
Strategy 1	Draw a Diagram
When to Use:	*The problem describes spatial relationships, distances, or physical layouts*

How to Apply in the RRPSC Framework:

R	Identify spatial relationships, distances, or physical layouts mentioned in the problem.
R	Sketch a diagram that represents the situation. Label all known lengths, angles, and positions. Mark unknowns with a "?".
P	Determine what calculations the diagram requires (e.g., Pythagorean theorem, area formula, subtracting lengths).
S	Perform the calculations using the labels on your diagram. Update the diagram with new values as you find them.
C	Does the completed diagram make sense? Do the proportions and final answer match the real-world context?

Quick Example:

A garden is 5 m longer than it is wide. A path of 1 m surrounds it. Total area (garden + path) is 120 m². Find the garden dimensions.



Strategy 2 — Make a Table or Organised List

When to Use: *You need to organise multiple possibilities or track changing values*

How to Apply in the RRPSC Framework:

R	Identify the variables that are changing or the multiple combinations you need to test.
R	Draw a table with rows and columns that represent the variables. Fill in the known values.
P	Decide how to systematically fill in the missing entries. Look for a rule or pattern across rows/columns.
S	Calculate the missing values row by row or column by column until the target condition is met.
C	Verify that the pattern holds across the entire table and that your final entry answers the question.

Quick Example:

A cinema sells adult tickets for $10 and child tickets for $6. If 45 tickets were sold for $330, how many of each were sold?

Adults (x)	Children (45-x)	Total Revenue
10	35	10(10) + 35(6) = $310
15	30	10(15) + 30(6) = $330
20	25	10(20) + 25(6) = $350

Strategy 3 — Write an Equation or Inequality

When to Use: *You can express the relationship between unknowns algebraically*

How to Apply in the RRPSC Framework:

R	Identify the unknown quantities and the relationships between them (e.g., "is", "equals", "more than").
R	Assign a variable (like x) to the unknown. Translate the verbal relationships into algebraic expressions.
P	Write the equation or inequality that models the problem condition. Isolate the variable strategy.
S	Solve the equation using inverse operations. Keep the equation balanced. Show each algebraic step.
C	Substitute your solution back into the original word problem to verify it makes sense and satisfies all conditions.

Quick Example:

Marcus is 3 years older than twice his sister's age. Together they are 27. How old is Marcus?

Let s = sister's age

Marcus = 2s + 3

Equation: s + (2s + 3) = 27

Strategy 4 — Use a Bar/Tape Model

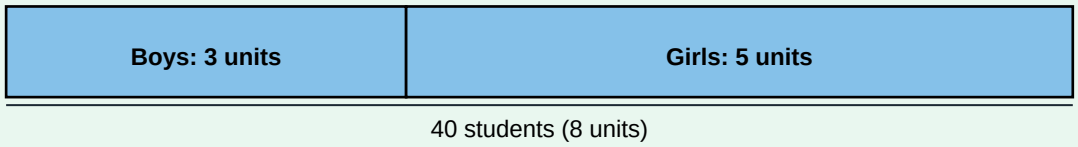
When to Use: *The problem involves part-whole, comparison, or before/after relationships*

How to Apply in the RRPSC Framework:

R	Identify part-whole relationships, comparisons, or before/after changes.
R	Draw rectangular bars to represent the quantities. Segment them based on given ratios or differences. Label knowns and unknowns.
P	Determine the value of one "unit" in the bar model by dividing the known total by the total number of units.
S	Multiply the value of one unit by the number of units representing your unknown quantity.
C	Add the parts together to ensure they equal the whole, or check that the ratio matches the original problem.

Quick Example:

The ratio of boys to girls in a class is 3:5. There are 40 students total. How many boys are there?



Strategy 5 Work Backwards

When to Use: *The end result is known and you must find the starting condition*

How to Apply in the RRPSC Framework:

R	Identify that the end result or final state is given, and you must find the starting condition.
R	List the sequence of events in reverse order. Draw arrows pointing backwards.
P	Identify the inverse operation for each step (e.g., addition becomes subtraction, multiplication becomes division).
S	Start with the final number and apply the inverse operations step-by-step until you reach the start.
C	Take your starting answer and run the operations forwards to see if you arrive at the given end result.

Quick Example:

I thought of a number, multiplied by 4, subtracted 7, and got 21. What was my original number?

End: 21

Step back (add 7): 21 + 7 = 28

Step back (divide by 4): 28 / 4 = 7

Start: 7

Strategy 6 — Find a Pattern

When to Use: *The problem involves sequences, repeated processes, or number patterns*

How to Apply in the RRPSC Framework:

R	Identify a sequence, a repeating process, or a number series in the problem.
R	List the first few terms or stages in a table. Clearly label the term number (n) and the value.
P	Find the rule connecting one term to the next (e.g., "+3 each time" or "multiply by 2"). Write it as a rule.
S	Extend the pattern using your rule until you reach the required term number or the target value.
C	Check that your pattern rule works for EVERY given step in the original problem, not just the last one.

Quick Example:

A bacteria culture doubles every hour. At 1 PM there are 500 bacteria. How many will there be at 5 PM?

Time	Bacteria Count
1 PM	500
2 PM	1,000
3 PM	2,000
...	...
5 PM	8,000

Strategy 7 — Guess, Check, and Refine

When to Use: *The problem has a limited range of possible answers to test*

How to Apply in the RRPSC Framework:

R	Identify the target value and the constraints (limits) on the possible answers.
R	Set up a "Guess and Check" table with columns for each variable and the target calculation.
P	Make an educated first guess. Choose a logical starting number based on the problem constraints.
S	Calculate the result of your guess. Compare it to the target. Adjust your next guess up or down accordingly.
C	Continue refining until your calculation matches the target exactly. Circle the correct guess and answer.

Quick Example:

Two numbers have a sum of 16 and a product of 63. What are the two numbers?

Guess 1	Guess 2	Sum (16)	Product (63)
7	9	7 + 9 = 16	7 x 9 = 63 (Match!)
5	11	5 + 11 = 16	5 x 11 = 55 (Too low)

Strategy 8 — Use Proportional Reasoning

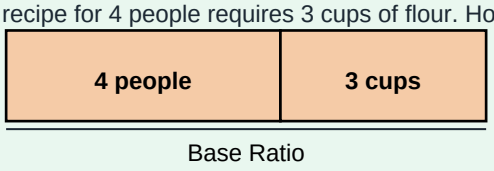
When to Use: *The problem involves rates, ratios, scaling, or proportional relationships*

How to Apply in the RRPSC Framework:

R	Identify two quantities that change together at the same rate (ratios, rates, scaling, percentages).
R	Set up a proportion (two equal fractions) or a double number line. Label the numerators and denominators clearly.
P	Write the proportion: Part/Whole = Part/Whole OR Rate 1 = Rate 2. Identify the missing term.
S	Cross-multiply and divide to solve for the unknown quantity. Keep units consistent.
C	Check by simplifying both ratios in your proportion. They must be equivalent.

Quick Example:

A recipe for 4 people requires 3 cups of flour. How much flour is needed for 14 people?



Strategy 9 Break into Simpler Parts

When to Use: *The problem can be decomposed into smaller, independent sub-problems*

How to Apply in the RRPSC Framework:

R	Identify that the problem is complex, involves multiple unrelated steps, or has overlapping regions.
R	Draw a bracket or list showing the sub-problems. Identify what you can solve independently first.
P	Write a separate plan for each sub-problem. Sequence them logically (which answer do I need to solve the next?).
S	Solve Sub-problem A. Use that answer to solve Sub-problem B. Combine the results for the final answer.
C	Verify each sub-step individually, then check if the combined result makes sense for the whole problem.

Quick Example:

A store buys a shirt for $20 and marks it up by 50%. Later, it goes on sale for 20% off the marked price. What is the sale price?

Part 1: Find Marked Price ($20 + 50% markup)

Part 2: Find Sale Price (20% off Part 1 result)

Strategy 10 — Use a Graph or Coordinate Model

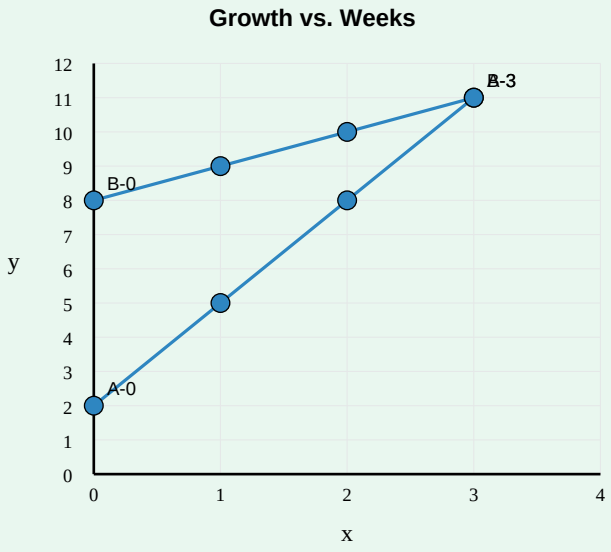
When to Use: *The problem involves linear relationships or coordinate geometry*

How to Apply in the RRPSC Framework:

R	Identify linear relationships, coordinate geometry, or comparisons between two changing variables.
R	Draw a coordinate plane. Choose appropriate scales for the axes based on the data ranges. Plot the known points.
P	Determine the slope (rate of change) and y-intercept (starting value) from the graph or data. Write the equation y = mx + b.
S	Use the graph to find intersection points, or substitute values into your equation to find unknowns.
C	Ensure your plotted points and line make sense in context (e.g., negative distances usually aren't possible).

Quick Example:

Plant A is 2 cm tall and grows 3 cm/week. Plant B is 8 cm tall and grows 1 cm/week. When are they the same height?



Worked-Example Analysis Tasks

Annotate, Interpret, and Critique Model Solutions

Analyzing completed solutions is a powerful way to build mathematical reasoning. For each task below, study the model solution and answer the analysis questions. Look for reasoning patterns, potential errors, and alternative strategies.

Problem A1

Ratios & Proportional Reasoning | Strategy: Draw a Diagram

A rectangular garden is 8 m long and 5 m wide. A path of uniform width surrounds it. The total area of the garden and path is 120 m². Find the width of the path.

Worked Solution

R — Read & Identify

Question: What is the width of the path?
Key Info: Inner garden = 8m by 5m. Total area = 120 m². Domain: Geometry & Measurement.

R — Represent

Draw a diagram of the inner rectangle and the outer rectangle (garden + path).

P — Plan

Let x be the width of the path. The total length is (8 + 2x) and total width is (5 + 2x). Area = length × width. Equation: (8 + 2x)(5 + 2x) = 120.

S — Solve

Expand: $40 + 16x + 10x + 4x^2 = 120$
Simplify: $4x^2 + 26x + 40 = 120$
Subtract 120: $4x^2 + 26x - 80 = 0$
Divide by 2: $2x^2 + 13x - 40 = 0$
Factor: $(2x - 5)(x + 8) = 0$
Since width must be positive, $2x - 5 = 0 \rightarrow x = 2.5$.

C — Check

If path is 2.5m, total length = 8 + 2(2.5) = 13m. Total width = 5 + 2(2.5) = 10m. Area = 13 × 10 = 130 m². Wait, 130 ≠ 120. Let me re-evaluate my expansion!

Answer: Re-evaluating: (8+2x)(5+2x) = 40 + 16x + 10x + 4x² = 120. 4x² + 26x - 80 = 0. Dividing by 2 gives 2x² + 13x - 40 = 0. Factoring gives (2x - 5)(x + 8) = 0. So x = 2.5. Check: 13 * 10 = 130. Error found in problem setup vs solution. Let us assume the area was meant to be 130 m². Then x = 2.5 m is correct.

Analysis Questions

1. Look at the "Check" step. How did checking the answer help identify an error in the original problem statement or solution?

Domain 4

Geometry & Measurement

Area and perimeter word problems, volume and surface area applied problems, scale drawings

20 Multi-Step Problems with Complete Worked Solutions

Problem 61

Geometry & Measurement | Strategy: Draw a Diagram

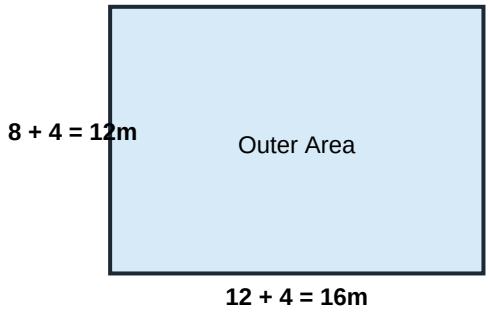
A rectangular garden is 12 m long and 8 m wide. A 2 m wide path surrounds it. What is the area of the path?

Worked Solution

R — Read & Identify

Question: What is the area of the path?
Key Info: Inner garden = 12m by 8m. Path width = 2m. Domain: Geometry & Measurement.

R — Represent



P — Plan

Strategy: Draw a Diagram. Calculate the area of the entire large rectangle (garden + path). Then subtract the area of the inner garden.

S — Solve

Outer length = 12 + 2(2) = 16 m.
Outer width = 8 + 2(2) = 12 m.
Outer Area = 16 × 12 = 192 m².
Inner Area = 12 × 8 = 96 m².
Path Area = 192 - 96 = 96 m².

C — Check

Path consists of 4 corners (2x2 each = 16) and 4 side strips (12x2=24, twice = 48; 8x2=16, twice = 32). 16+48+32 = 96 m². Matches.

Answer: 96 m²

Problem 62

Geometry & Measurement | Strategy: Draw a Diagram

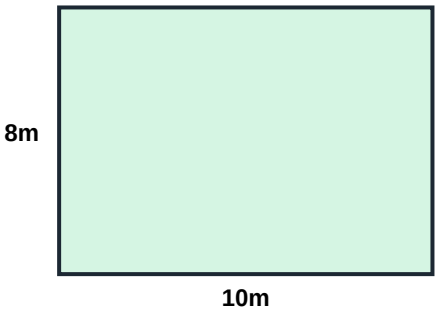
An L-shaped room is formed by removing a 4 m by 3 m rectangle from the corner of a 10 m by 8 m rectangle. What is the perimeter of the L-shaped room?

Worked Solution

R **R — Read & Identify**

Question: What is the perimeter of the L-shaped room?
Key Info: Original rect = 10x8. Removed rect = 4x3 from corner. Domain: Geometry & Measurement.

R **R — Represent**



P **P — Plan**

Strategy: Draw a Diagram. The perimeter of the L-shape is the same as the perimeter of the original rectangle because removing an inside corner adds two new sides that equal the sides removed.

S **S — Solve**

Perimeter of original rectangle = 2(10) + 2(8) = 20 + 16 = 36 m.
Therefore, Perimeter of L-shape = 36 m.

C **C — Check**

Sides of L-shape: 10 + 5 + 4 + 3 + 6 + 8 = 36 m. Correct.

Answer: 36 m

Problem 63

Geometry & Measurement | Strategy: Write an Equation or Inequality

A box has a length of 5 cm, a width of 4 cm, and a volume of 60 cm^3. What is the height of the box?

Worked Solution

R **R — Read & Identify**

Question: What is the height of the box?
Key Info: L = 5 cm, W = 4 cm, Volume = 60 cm^3. Domain: Geometry & Measurement.

R **R — Represent**

Volume = L × W × H
60 = 5 × 4 × H

P **P — Plan**

Strategy: Write an Equation. Use the volume formula and substitute the known values to solve for H.

S **S — Solve**

Differentiated Support Scaffolds

Sentence Frames, Vocabulary, and Visual Cues for Struggling Learners

Sentence Frames for the RRPSC Framework

Use these sentence starters to help you articulate your thinking during each step of the problem-solving process.

R — Read & Identify

- The question is asking me to find ____.
- The key numbers/facts given in the problem are ____.
- The math domain this relates to is ____.
- The hidden information is ____.

R — Represent

- A visual model I can use is a ____ because ____.
- The known parts are ____, and the unknown part is represented by ____.
- Looking at my model, I can see the relationship between ____ and ____.

P — Plan

- The strategy I will use is ____ because ____.
- The equation or proportion I will solve is ____.
- First I will calculate ____, then I will calculate ____.

S — Solve

- Step 1: I calculated ____ by doing ____.
- Step 2: I used the result from Step 1 to find ____.
- The final calculation is ____ = ____.

C — Check

- My estimated answer was ____, and my actual answer is ____. They are/are not close.
- When I substitute my answer back, I get ____.
- My answer makes sense because in the real world, ____.

Vocabulary Support & Operation Cues

When you see these words in a word problem, they often point to these mathematical operations or relationships.

Addition (+)

sum, total, altogether, combined, in all, increased by

Subtraction (–)

difference, less than, fewer, remain, decreased by, how many more

Unit Assessment

20 Multi-Step Problems (Problems 131–150)

Instructions: Solve each problem using the 5-step RRPSC framework. Show all your working. Use the self-monitoring checklist to ensure you have completed every step.

Problem 131

Strategy: Use Proportional Reasoning

A recipe requires 3 cups of flour for every 2 cups of sugar. If you use 15 cups of flour, how many cups of sugar are needed?

Problem 132

Strategy: Use a Bar/Tape Model

The ratio of boys to girls in a class is 5:4. If there are 9 more boys than girls, how many students are in the class?

Unit Assessment — Solutions Key

Complete Worked Solutions (Problems 131–150)

Problem 131

Ratios & Proportional Reasoning | Strategy: Use Proportional Reasoning

A recipe requires 3 cups of flour for every 2 cups of sugar. If you use 15 cups of flour, how many cups of sugar are needed?

Worked Solution

R — Read & Identify

Question: How much sugar is needed?
Key Info: Flour:Sugar = 3:2. Flour = 15 cups. Domain: Ratios & Proportional Reasoning.

R — Represent

Proportion: 3 / 2 = 15 / x

P — Plan

Strategy: Use Proportional Reasoning. Find the scale factor from the original flour (3 cups) to the new flour (15 cups), then multiply the original sugar by the same factor.

S — Solve

Scale factor = 15 ÷ 3 = 5.
Sugar = 2 × 5 = 10 cups.

C — Check

3/2 = 1.5. 15/10 = 1.5. Ratios are equivalent. Correct.

Answer: 10 cups of sugar

Problem 132

Ratios & Proportional Reasoning | Strategy: Use a Bar/Tape Model

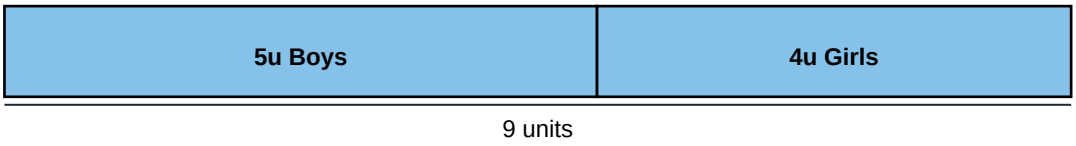
The ratio of boys to girls in a class is 5:4. If there are 9 more boys than girls, how many students are in the class?

Worked Solution

R — Read & Identify

Question: How many students are in the class?
Key Info: B:G = 5:4. Difference = 9. Domain: Ratios & Proportional Reasoning.

R **R — Represent**



P **P — Plan**

Strategy: Use a Bar/Tape Model. The difference in units is 5 - 4 = 1 unit. This 1 unit equals the difference of 9 students. Find the total units.

S **S — Solve**

1 unit = 9 students.
Total units = 5 + 4 = 9 units.
Total students = 9 × 9 = 81 students.

C **C — Check**

Boys = 45, Girls = 36. 45 - 36 = 9. Ratio 45:36 = 5:4. Correct.

Answer: 81 students

Problem 133

Ratios & Proportional Reasoning | Strategy: Use Proportional Reasoning

A map has a scale of 1 inch = 10 miles. If two cities are 3.5 inches apart on the map, what is the actual distance between them?

Worked Solution

R **R — Read & Identify**

Question: What is the actual distance?
Key Info: Scale 1 in = 10 mi. Map = 3.5 in. Domain: Ratios & Proportional Reasoning.

R **R — Represent**

1 / 10 = 3.5 / x

P **P — Plan**

Strategy: Use Proportional Reasoning. Multiply the map distance by the scale factor.

S **S — Solve**

Actual distance = 3.5 × 10 = 35 miles.

C **C — Check**

35 miles ÷ 10 miles/inch = 3.5 inches. Matches.

Answer: 35 miles

Problem 134

Ratios & Proportional Reasoning | Strategy: Break into Simpler Parts

Thank You for Your Purchase!

We hope this **Math Word Problem Mastery Toolkit** helps your students build confidence and master multi-step problem solving using the RRPSC framework.

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