UM Printables

Premium Mathematics Resources

GEOMETRY

PROOF WRITING UNIT

Step-by-Step Scaffolds

Grades 9 - 10 | 10 Lessons | 40 Practice Problems

Two-Column Proof Structure and Logic
Angle Relationships and Linear Pairs
Parallel Lines and Transversal Proofs
Triangle Congruence: SSS, SAS, ASA, AAS, HL
CPCTC and Isosceles Triangle Proofs
Quadrilateral Property Proofs

Includes:
3 Scaffold Levels per Problem (120 Practice Proofs Total)
10 Completed Proof Exemplars | Proof Vocabulary Card Set
Common Proof Error Analysis Activity | Unit Test with Answer Key

Table of Contents

How to Use This Resource

This resource is designed to systematically build student confidence in geometric proof writing through a gradual release model. Each lesson follows a consistent structure that moves students from guided practice to independent mastery.

Lesson Structure

Each of the 10 lessons includes a concept overview with key definitions and theorems, geometric diagrams, one or more completed proof exemplars, and practice problems at three scaffold levels:

Scaffold Level	Description	Student Role
Level 1	Heavily Scaffolded	All statements are provided. Students fill in the reasons using a word bank.
Level 2	Partially Scaffolded	Some rows are completed and others are blank. Students determine the missing statements and reasons.
Level 3	Independent	Only the Given and Prove are provided. Students write the entire proof from scratch.

Suggested Pacing

Each lesson is designed for one to two class periods. Begin with the concept overview and exemplar, then have students work through Level 1 problems. Level 2 problems are ideal for collaborative group work. Level 3 problems serve as independent practice or formative assessments.

Diagrams

Geometric diagrams are included throughout the resource to help students visualize the relationships in each proof. Encourage students to mark their diagrams with given information before starting a proof. This habit strengthens spatial reasoning and helps identify the correct congruence postulates or theorems.

Vocabulary Cards

The 30 vocabulary cards can be printed, cut, and used as flashcards or reference tools. Encourage students to keep their card set accessible during proof writing.

Proof Vocabulary Card Set

30 essential terms for geometry proof writing. Print, cut apart, and use as flashcards or reference tools.

Proof

A logical argument that uses definitions, postulates, theorems, and previously proven statements to show that a given statement is true.

Two-Column Proof

A proof format with numbered statements in the left column and corresponding reasons in the right column. Each reason justifies the statement on the same line.

Given

Information accepted as true at the start of a proof. The Given is the starting point for all logical reasoning.

Prove

The statement that must be shown to be true through logical reasoning. This is the goal of the proof.

Statement

A mathematical claim written in the left column of a two-column proof. Each statement follows logically from previous statements or the given.

Reason

The justification for a statement. Valid reasons include: Given, definitions, postulates, theorems, and algebraic properties.

Theorem

A statement that has been proven true using definitions, postulates, and previously established theorems.

Postulate (Axiom)

A statement accepted as true without proof. Postulates are the foundational building blocks of a mathematical system.

Congruent (≅)

Having the same size and shape. Segments are congruent if they have the same length. Angles are congruent if they have the same measure.

Vertical Angles

Two non-adjacent angles formed by two intersecting lines. Vertical angles are always congruent.

Complementary Angles

Two angles whose measures have a sum of 90°.

Supplementary Angles

Two angles whose measures have a sum of 180°.

Linear Pair

Two adjacent angles whose non-common sides form a straight line. A linear pair is always supplementary.

Transversal

A line that intersects two or more other lines at distinct points.

Corresponding Angles

Angles in the same relative position at each intersection of a transversal with two lines. If lines are parallel, they are congruent.

Alternate Interior Angles

Angles on opposite sides of a transversal and between the two lines. If lines are parallel, they are congruent.

Same-Side Interior Angles

Angles on the same side of a transversal and between the two lines. If lines are parallel, they are supplementary.

SSS Congruence Postulate

If three sides of one triangle are congruent to three sides of another, then the triangles are congruent.

SAS Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another, then the triangles are congruent.

ASA Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another, then the triangles are congruent.

AAS Congruence Theorem

If two angles and a non-included side of one triangle are congruent to the corresponding parts of another, then the triangles are congruent.

HL Congruence Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another, then the triangles are congruent.

CPCTC

Corresponding Parts of Congruent Triangles are Congruent. After proving triangles congruent, all corresponding parts are also congruent.

Reflexive Property

Any quantity is equal (or congruent) to itself. Used when two triangles share a common side or angle.

Symmetric Property

If a = b, then b = a. The order of a congruence or equality statement can be reversed.

Transitive Property

If a = b and b = c, then a = c. Congruence can be chained through a common element.

Substitution Property

A quantity may be substituted for its equal in any expression or equation.

Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Parallelogram

A quadrilateral with both pairs of opposite sides parallel. Opposite sides and angles are congruent; diagonals bisect each other.

Lesson 1: Introduction to Two-Column Proofs and Logic

Understanding the structure of statements and reasons

Algebraic properties and basic logical reasoning

Key Concepts

A **two-column proof** is a logical argument organized into numbered statements and reasons. The left column contains the **statements** (the mathematical claims), and the right column contains the **reasons** (the justifications). Every statement must be supported by a valid reason.

Valid reasons include: Given information, Definitions, Postulates (axioms), Theorems (previously proven statements), and Algebraic Properties of Equality.

Property	Statement
Addition Property of Equality	If a = b, then a + c = b + c.
Subtraction Property of Equality	If a = b, then a - c = b - c.
Multiplication Property of Equality	If a = b and c ≠ 0, then ac = bc.
Division Property of Equality	If a = b and c ≠ 0, then a/c = b/c.
Reflexive Property	a = a (or a ≅ a)
Symmetric Property	If a = b, then b = a.
Transitive Property	If a = b and b = c, then a = c.
Substitution Property	If a = b, then b can replace a in any expression.
Distributive Property	a(b + c) = ab + ac

Completed Proof Exemplar

Key Idea: Start with the Given statement. Work step-by-step toward the Prove statement. Each step must follow logically from the previous one. The last statement in your proof should always match the Prove statement exactly.

Given: 2(x + 5) = 30

Prove: x = 10

Statements	Reasons
1. 2(x + 5) = 30	1. Given
2. 2x + 10 = 30	2. Distributive Property
3. 2x = 20	3. Subtraction Property of Equality
4. x = 10	4. Division Property of Equality

Practice Problems

Problem 1

Given: 3x - 7 = 20

Prove: x = 9

Level 1: Heavily Scaffolded -- Fill in the Reasons

Reasons Word Bank: *Addition Property of Equality | Division Property of Equality*

Statements	Reasons
1. 3x - 7 = 20	1. Given
2. 3x = 27	2. ______________________________
3. x = 9	3. ______________________________

Problem 1

Given: 3x - 7 = 20

Prove: x = 9

Level 2: Partially Scaffolded -- Complete the Missing Steps

There are 3 steps in this proof. Complete the missing rows.

Statements	Reasons
1. 3x - 7 = 20	1. Given
2. ______________________________	2. ______________________________
3. x = 9	3. Division Property of Equality

Problem 1

Given: 3x - 7 = 20

Prove: x = 9

Level 3: Independent -- Write the Full Proof

Write the complete proof. Plan your steps before writing.

Statements	Reasons
1. ________________________________	1. ________________________________
2. ________________________________	2. ________________________________
3. ________________________________	3. ________________________________

Problem 2

Given: AB = CD, CD = EF

Prove: AB = EF

Level 1: Heavily Scaffolded -- Fill in the Reasons

Reasons Word Bank: *Given | Transitive Property of Equality*

Statements	Reasons
1. AB = CD	1. Given
2. CD = EF	2. ______________________________
3. AB = EF	3. ______________________________

Problem 2

Given: AB = CD, CD = EF

Prove: AB = EF

Level 2: Partially Scaffolded -- Complete the Missing Steps

There are 3 steps in this proof. Complete the missing rows.

Statements	Reasons
1. AB = CD	1. Given
2. ______________________________	2. ______________________________
3. AB = EF	3. Transitive Property of Equality

Problem 2

Given: AB = CD, CD = EF

Prove: AB = EF

Level 3: Independent -- Write the Full Proof

Write the complete proof. Plan your steps before writing.

Statements	Reasons
1. ___________________________________	1. ___________________________________
2. ___________________________________	2. ___________________________________
3. ___________________________________	3. ___________________________________

Problem 3

Given: m∠A = 45°, m∠B = 45°

Prove: ∠A ≅ ∠B

Level 1: Heavily Scaffolded -- Fill in the Reasons

Reasons Word Bank: *Given | Transitive Property of Equality | Definition of Congruent Angles*

Statements	Reasons
1. m∠A = 45°	1. Given
2. m∠B = 45°	2. ________________________
3. m∠A = m∠B	3. ________________________
4. ∠A ≅ ∠B	4. ________________________

Problem 3

Given: m∠A = 45°, m∠B = 45°

Prove: ∠A ≅ ∠B

Level 2: Partially Scaffolded -- Complete the Missing Steps

There are 4 steps in this proof. Complete the missing rows.

Statements	Reasons
1. m∠A = 45°	1. Given
2. m∠B = 45°	2. Given
3. ________________________	3. ________________________
4. ∠A ≅ ∠B	4. Definition of Congruent Angles

Problem 3

Given: m∠A = 45°, m∠B = 45°

Prove: ∠A ≅ ∠B

Level 3: Independent -- Write the Full Proof

Write the complete proof. Plan your steps before writing.

Statements	Reasons
1. ________________________________	1. ________________________________
2. ________________________________	2. ________________________________
3. ________________________________	3. ________________________________
4. ________________________________	4. ________________________________

Lesson 2: Angle Relationships

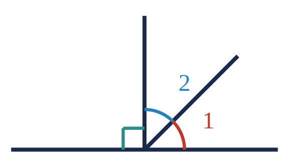
Complementary, supplementary, vertical angles, and linear pairs

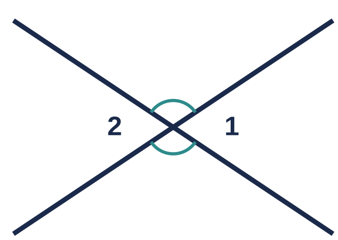
Using angle postulates and theorems in proofs

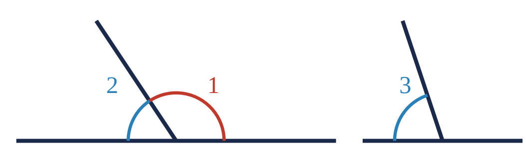
Key Concepts

Understanding angle relationships is essential for writing geometric proofs. These relationships provide the reasons that connect statements logically. Memorize the definitions and theorems in this section, as they are used frequently throughout the rest of the unit.

Angle Pair Diagrams







Relationship	Definition / Theorem
Complementary Angles	Two angles whose measures sum to 90°.
Supplementary Angles	Two angles whose measures sum to 180°.

Lesson 3: Parallel Lines and Transversals

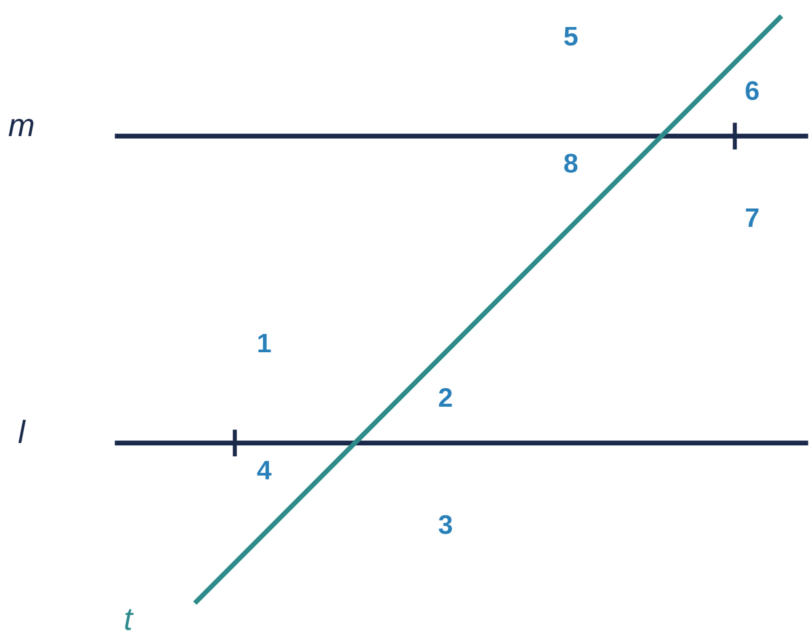
Corresponding, alternate interior, and same-side interior angles

Proving angle relationships using parallel line postulates and theorems

Key Concepts

When a transversal intersects two parallel lines, special angle relationships are formed. These relationships are guaranteed by postulates and theorems, making them powerful tools in proofs. Pay careful attention to the distinction between postulates (accepted without proof) and theorems (proven from postulates).

Parallel Lines Cut by a Transversal



Angles 1 and 5 are corresponding; Angles 4 and 6 are alternate interior; Angles 4 and 5 are same-side interior.

Postulate / Theorem	Statement
Corresponding Angles Postulate	If two parallel lines are cut by a transversal, then corresponding angles are congruent.
Alternate Interior Angles Theorem	If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Lesson 4: Triangle Congruence: SSS and SAS

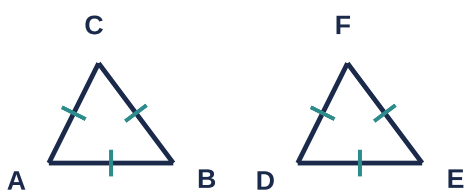
Side-Side-Side and Side-Angle-Side postulates

Proving triangles congruent using sides and included angles

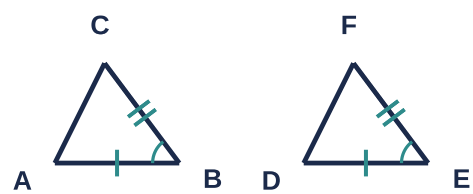
Key Concepts

Two triangles are congruent if their corresponding sides and angles are congruent. However, it is not necessary to prove all six pairs congruent. The SSS and SAS postulates provide shortcuts based on just three pairs of congruent corresponding parts. When writing the congruence statement, always match corresponding vertices in the same order.

SSS and SAS Congruence Diagrams



SSS: Three pairs of congruent sides



SAS: Two sides and the included angle

Postulate	Required Congruent Parts	Key Detail
SSS Congruence Postulate	Three pairs of congruent sides	All three sides of one triangle must be congruent to the corresponding three sides of the other.
SAS Congruence Postulate	Two pairs of congruent sides and one pair of congruent included angles	The congruent angle must be included between the two congruent sides. The order matters: Side-Angle-Side.

Key Idea: When triangles share a common side, use the Reflexive Property of Congruence to establish that the shared side is congruent to itself. This is a crucial step in many triangle congruence proofs.

Lesson 5: Triangle Congruence: ASA and AAS

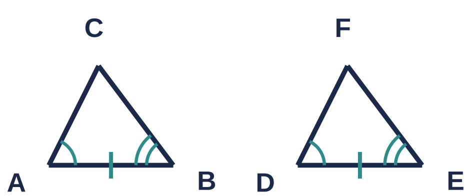
Angle-Side-Angle postulate and Angle-Angle-Side theorem

Proving triangles congruent using angles and included or non-included sides

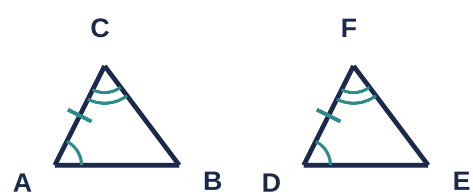
Key Concepts

The ASA and AAS criteria provide two more methods for proving triangle congruence. Note that ASA is a postulate (accepted without proof), while AAS is a theorem (proven using the angle sum of a triangle and ASA). In practice, both are used as valid reasons in proofs. The critical distinction is whether the side is included between the two angles (ASA) or non-included (AAS).

ASA and AAS Congruence Diagrams



ASA: Two angles and the included side



AAS: Two angles and a non-included side

Postulate / Theorem	Required Congruent Parts	Key Detail
ASA Congruence Postulate	Two pairs of congruent angles and one pair of congruent included sides	The congruent side must be included between the two congruent angles. Angle-Side-Angle.
AAS Congruence Theorem	Two pairs of congruent angles and one pair of congruent non-included sides	The congruent side is not between the two congruent angles. It is adjacent to only one of the angles.

Key Idea: Vertical angles are frequently used in ASA and AAS proofs. When two lines intersect, the vertical angles formed are congruent, providing one of the angle pairs needed.

Lesson 6: Triangle Congruence: HL and Right Triangles

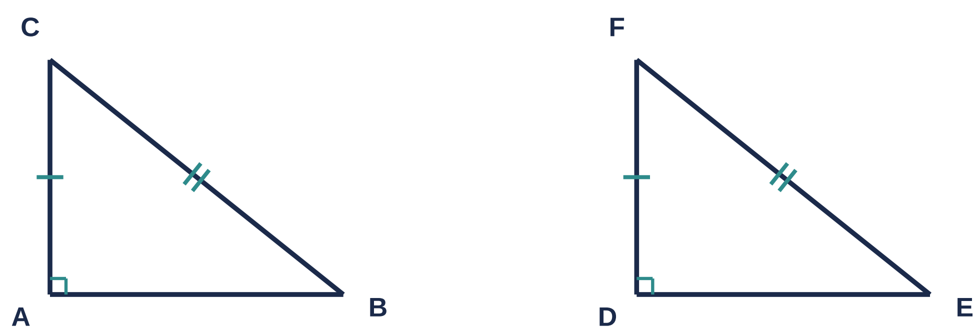
Hypotenuse-Leg theorem for right triangle congruence

Using the special HL shortcut exclusive to right triangles

Key Concepts

The Hypotenuse-Leg (HL) Theorem is a special congruence criterion that applies only to right triangles. Because the Pythagorean Theorem governs the relationship between the sides of a right triangle, knowing the hypotenuse and one leg are congruent is sufficient to prove the triangles congruent. This would not work for non-right triangles (there is no "SSA" shortcut).

HL Congruence Diagram



HL: Right triangles with congruent hypotenuses and one pair of congruent legs

Term / Theorem	Definition / Statement
Right Triangle	A triangle that contains exactly one right angle (90°). The side opposite the right angle is the hypotenuse.
Hypotenuse	The longest side of a right triangle, always opposite the right angle. It is the side that is not a leg.
Leg	One of the two sides of a right triangle that form the right angle. Every right triangle has exactly two legs.
HL Congruence Theorem	If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the two triangles are congruent.

Lesson 7: CPCTC

Corresponding Parts of Congruent Triangles are Congruent

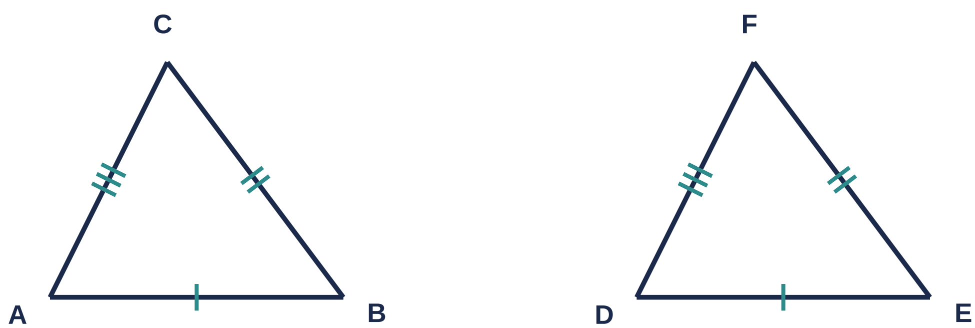
Using triangle congruence to prove other parts congruent

Key Concepts

Once two triangles have been proven congruent, all of their corresponding parts (sides and angles) are also congruent. The abbreviation **CPCTC** stands for **C**orresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent. CPCTC is used as a reason in a proof after the triangles have already been proven congruent by SSS, SAS, ASA, AAS, or HL.

Key Idea: CPCTC can only be used AFTER you have proven the triangles congruent. First, prove the triangles congruent using a postulate or theorem. Then, use CPCTC to prove that specific corresponding sides or angles are congruent.

Completed Proof Exemplar



Given: AB ≅ DE, BC ≅ EF, AC ≅ DF

Prove: ∠A ≅ ∠D

Statements	Reasons
1. AB ≅ DE	1. Given
2. BC ≅ EF	2. Given
3. AC ≅ DF	3. Given

Answer Key

Completed proofs for all practice problems.

Problem 1

Given: 3x - 7 = 20

Prove: x = 9

Statements	Reasons
1. 3x - 7 = 20	1. Given
2. 3x = 27	2. Addition Property of Equality
3. x = 9	3. Division Property of Equality

Problem 2

Given: AB = CD, CD = EF

Prove: AB = EF

Statements	Reasons
1. AB = CD	1. Given
2. CD = EF	2. Given
3. AB = EF	3. Transitive Property of Equality

Problem 3

Given: m∠A = 45°, m∠B = 45°

Prove: ∠A ≅ ∠B

Statements	Reasons
1. m∠A = 45°	1. Given
2. m∠B = 45°	2. Given
3. m∠A = m∠B	3. Transitive Property of Equality
4. ∠A ≅ ∠B	4. Definition of Congruent Angles

Problem 40

Given: WXYZ is a rectangle

Prove: WY ≅ XZ

Statements	Reasons
1. WXYZ is a rectangle	1. Given
2. WY ≅ XZ	2. Diagonals of a rectangle are congruent