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PROJECT-BASED LEARNING • 5-DAY UNIT

Exponential Growth & Decay

Real-World PBL | Compound Interest, Population & Viral Spread — with ELL Support

Algebra 1 | Grades 9–10 | CCSS-Aligned | ELL Scaffolds

A 5-Day Investigation Through Four Real Datasets

Day 1 • Patterns That Multiply — Foundations of exponential functions
Day 2 • Compound Interest — $1,000 savings over 30 years at real Fed rates
Day 3 • U.S. Population Growth — Census data, 1790–2020
Day 4 • Bacterial Growth & Viral Spread — E. coli doubling + SIR-lite epidemic
Day 5 • Student Choice Board — Model, graph, and present a data story

SECTION 1

Unit Overview & Pacing Guide

Welcome to a five-day investigation of one of mathematics' most powerful ideas: ***exponential change****. Students often first meet exponential functions as an abstract symbol-pushing exercise — memorize the formula, plug in numbers, move on. This unit takes a different path. It grounds every concept in a real, dated, verifiable dataset, so that students experience exponential reasoning the way scientists, economists, and public-health researchers actually use it: as a tool for making sense of a world where things multiply rather than add.*

Across the five days, students investigate four connected scenarios. They begin with a savings account whose $1,000 principal grows for thirty years under interest rates drawn from real Federal Reserve history. They then examine two centuries of United States census data, watching a young nation multiply through immigration, westward expansion, and industrial growth. They turn to the laboratory, where a single bacterium becomes millions overnight. And they end with a simulation of an epidemic, using a simplified SIR model whose parameters come from historical outbreak data. By the fifth day, students choose one of these scenarios, build their own model, and present a data story to their peers.

Throughout, three threads run together: **mathematical modeling** (translating real situations into equations and back again), **data literacy** (reading tables and graphs critically, recognizing where models fit and where they break down), and **language development** (using academic English precisely, with explicit scaffolds for English Language Learners). The ELL scaffolds are not optional add-ons; they are the architecture of every lesson, and they benefit every student.

Five-Day Pacing at a Glance

The table below summarizes the daily arc. Each day includes a launch, direct instruction with a worked example, guided practice, partner discourse using structured sentence frames, and a formative check. Total instructional time assumes a 55-minute period; the unit is easily compressed to 3 days or extended to 7 by combining or expanding the practice sets.

Day	Focus	Essential Question	Embedded ELL Scaffolds
1	Foundations of Exponential Functions	How is "growing by a factor" different from "growing by a constant amount"?	Vocab pre-teach cards; visual concept anchors; sentence frames for describing patterns
2	Compound Interest (Savings Account)	Why does a 4% interest rate more than triple $1,000 in 30 years?	Vocabulary cards (principal, compounding); real Fed rate table; partner-talk stems
3	U.S. Population Growth (1790–2020)	When does an exponential model fit census data well — and when does it fail?	Census-data investigation; linear-vs-exponential comparison frames; home-language partner talk

For English Language Learners, the unit deliberately engages all four language domains — listening, speaking, reading, and writing — through integrated content-and-language instruction. The table maps each ELD standard cluster to the specific scaffolds embedded in the unit.

WIDA ELD Standard	Focus	Embedded in This Unit
ELD-LS.1-3 (Listening/Speaking)	Participate in collaborative conversations; express ideas; negotiate meaning.	Partner-talk stems (Sec. 5), structured discourse during Days 1–4, peer feedback Day 5.
ELD-R.1-3 (Reading)	Interpret grade-level informational text; identify key ideas; analyze language.	Vocabulary cards (Sec. 4), real-data tables, visual concept anchors (Sec. 6).
ELD-W.1-3 (Writing)	Compose informational/explanatory texts; use academic vocabulary; construct claims.	Data-story template with sentence starters (Day 5), exit tickets, written interpretations.
ELD-LC (Language Conventions)	Use grammatical structures and vocabulary appropriate to discipline.	Sentence frames that foreground conditional ("If ____, then ____") and comparative ("more than", "less than") structures.
ELD-DC.C1 (Disciplinary Concept: Math)	Communicate mathematical reasoning, including justifying, comparing, and generalizing.	Justify-Frame stems in Sec. 5; Day 5 presentation rubric rewards reasoning language.

A Note on the "Math-as-Language" Approach

Exponential functions are linguistically demanding. Words like *compounded*, *doubling time*, *asymptote*, and *rate* have everyday meanings that overlap imperfectly with their mathematical meanings. The sentence frames in Section 5 are not crutches; they are **apprenticeship structures** that let students practice the discourse of mathematics before they are expected to produce it independently. Research from the SIOP Model (Echevarría, Vogt & Short, 2017) and WIDA (2020) consistently shows that structured oral practice accelerates both content mastery and language development for all students — not only designated ELLs.

SECTION 4

Vocabulary Cards

Display these four high-leverage vocabulary cards before introducing each new concept. Each card includes the English term, a phonetic pronunciation guide, a student-friendly definition, a sample sentence that uses the term in a lesson-relevant context, the linked visual anchor, a quick-reference formula, and a "common error to avoid" tip that pre-empts the most frequent student misconception. Print at 110% scale for classroom wall display, or distribute as a printed reference sheet for student notebooks.

Exponential Growth

/ ex-po-NEN-shul GROHTH /

Definition: A pattern of increase in which a quantity grows by multiplying by the same factor (greater than 1) during each equal time interval. The general form is $y = a \cdot b^t$, where a is the starting value and $b > 1$ is the growth factor.

Example: A savings account that pays 4% interest every year shows exponential growth: the balance grows by a factor of 1.04 each year.

Visual anchor: *An upward-curving line that gets steeper as t increases (see Anchor 1).*

QUICK REFERENCE

FORMULA

$$y = a \cdot b^t \quad (b > 1)$$

COMMON ERROR TO AVOID

Students often confuse "grows by 4%" with "multiplied by 4". A 4% growth rate means b = **1.04**, not b = 4. The growth factor is always (1 + r), where r is the rate as a decimal.

Exponential Decay

/ ex-po-NEN-shul dih-KAY /

Definition: A pattern of decrease in which a quantity shrinks by multiplying by the same factor (between 0 and 1) during each equal time interval. The general form is $y = a \cdot b^t$, where $0 < b < 1$ is the decay factor. The amount approaches zero but never reaches it.

Example: A medicine's concentration in the blood shows exponential decay: every hour, 25% of the remaining amount is removed.

Visual anchor: *A downward-curving line that flattens out as it approaches the t-axis (an asymptote).*

QUICK REFERENCE

FORMULA

$$y = a \cdot b^t \quad (0 < b < 1)$$

COMMON ERROR TO AVOID

Do not subtract the decay rate from the amount. If 25% is removed each hour, the decay factor is b = **0.75**, not b = –0.25. The amount left is always multiplied by a positive number less than 1.

SECTION 5

Sentence Frames & Discourse Stems

Sentence frames are the bridge between mathematical thinking and mathematical discourse. They give students the syntactic structure they need to express ideas precisely, without doing the mathematical thinking for them. Use the frames below during partner talk, whole-class discussion, exit tickets, and the Day 5 presentation. Project the relevant frame on the board, model its use once, then give students 60 seconds of silent think time before they speak.

Frames for Describing Patterns

Use these frames when students are examining a table, graph, or real-world situation and identifying its mathematical structure.

- *I notice that as the time increases by ____, the amount ____.*
- *When the time increases by ____, the amount grows by a factor of ____.*
- *The initial value (y-intercept) is ____, which represents ____ in this situation.*
- *The growth factor is ____, which means that every ____, the amount is multiplied by ____.*
- *This graph shows (growth / decay) because ____.*
- *Looking at the table, I see that the values are (increasing / decreasing) by (a constant amount / a constant factor) ____.*

Frames for Comparing Models

Use these frames when students compare two functions (e.g., linear vs. exponential, or two different exponential models), or when they compare a model's predictions to actual data.

- *The exponential model fits better than the linear model because ____.*
- *Both models predict that ____, but they differ in ____.*
- *The actual data (agrees / disagrees) with my model at time t = ____. The difference is ____.*
- *Compared to the slow-growth scenario, the fast-growth scenario produces ____ by year ____.*
- *My model overestimates the actual value by ____, which suggests that ____.*
- *A limitation of my model is that it does not account for ____.*

Frames for Making Predictions

- *Using my model A(t) = ____, I predict that at t = ____, the value will be ____.*
- *Because the growth factor is ____, after ____ time periods the amount will be (multiplied / divided) by ____.*
- *To find the doubling time, I set* $A(t) = 2 \cdot A_0$ *and solved for t. I got t = ____.*
- *If the trend continues, in ____ years the amount will reach ____.*

SECTION 6

Visual Concept Anchors

These three full-color diagrams are the visual reference points for the entire unit. Print them on tabloid paper and post them in the classroom, or project them as students work. When a student is stuck, direct them to the appropriate anchor rather than re-explaining the concept verbally — the diagram does the cognitive work.

Anchor 1 — Growth vs. Decay, and Linear vs. Exponential

This two-panel anchor answers the two most common conceptual questions in the unit: **What does exponential growth look like, and how is it different from decay?** (left panel); and **Why does exponential eventually beat linear, even when linear starts out ahead?** (right panel). The annotated features — asymptote, y-intercept, growth factor, crossover point — are the same features students will identify in every real-data graph throughout the unit.

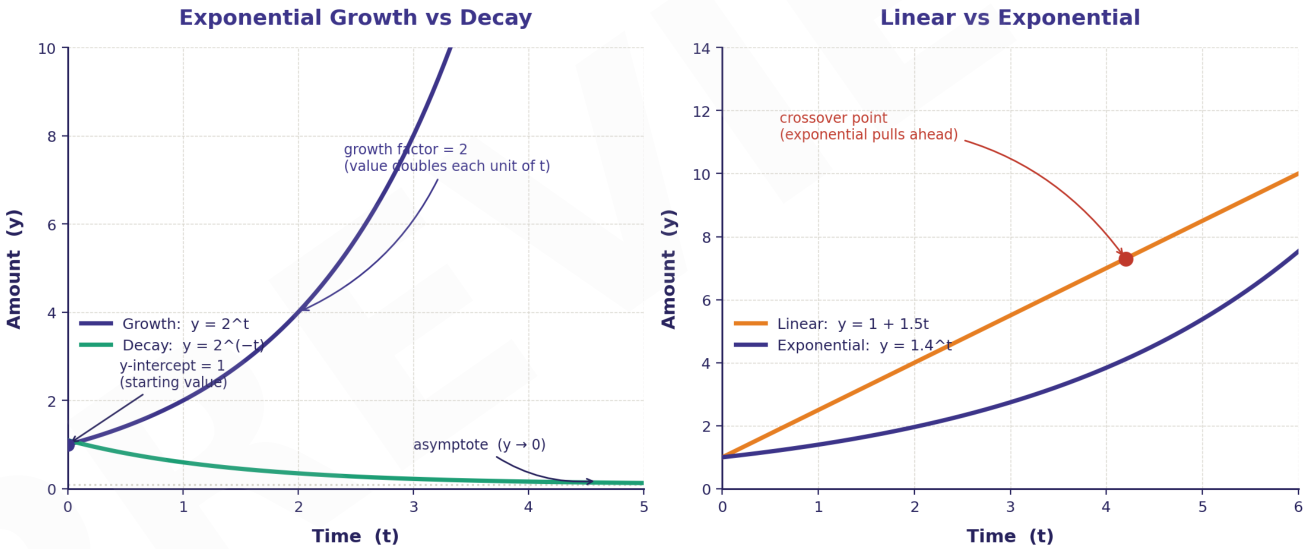

Figure 1. The two foundational shapes of the unit. Left: $y = 2^t$ (growth, purple) and $y = 2^{-t}$ (decay, teal). Right: linear $y = 1 + 1.5t$ (orange) is overtaken by exponential $y = 1.4^t$ (purple) near $t \approx 4.2$.

Anchor 2 — Doubling Time

The doubling time is the heartbeat of an exponential growth model. This anchor shows that doubling time is **constant**: no matter where you start on the curve, the amount doubles after the same interval. This is the property that distinguishes exponential growth from all other growth patterns and that makes the model so useful — and so dangerous, when applied to epidemics or debt.

DAY 1 LESSON

Patterns That Multiply — Foundations

Lesson at a Glance

Time: 55 minutes | **Focus:** Distinguishing multiplicative from additive growth; introducing $A(t) = A_0 \cdot b^t$
Essential Question: How is "growing by a factor" different from "growing by a constant amount"?
Materials: Visual Anchor 1 (Sec. 6), vocabulary cards for "exponential growth" and "growth factor", printed Day 1 packet, calculator per student
ELL Scaffolds Active: Vocab pre-teach (5 min), Describe-frame stems during partner talk, visual anchor on display throughout

Launch — The Legend of the Chessboard (8 min)

Open by telling the legend of the wise man who asked the king for a seemingly modest reward: one grain of wheat on the first square of a chessboard, two on the second, four on the third, and so on — doubling on every square. The king agrees, thinking the request trivial. The punchline: by the 64th square, the king owes more grains of wheat than have ever been harvested in human history — approximately 18.4×10^{18} grains.

Project the following table on the board. Do not announce the pattern. Instead, ask: *"What do you notice? What do you wonder?"*

Square #	Grains of wheat		Square #	Grains of wheat
1	1		7	64
2	2		8	128
3	4		9	256
4	8		10	512
5	16		11	1,024
6	32		12	2,048

After 60 seconds of silent observation, use the **Describe-frame** sentence stem on the board:

- *"I notice that as the square number increases by ____, the number of grains ____."*

Listen for the word **"doubles"**. When a student uses it, name it explicitly: *"You said the number doubles. In mathematics, we call this **exponential growth**. The growth factor is 2."* Display the vocabulary card for *Exponential Growth*.

Mini-Lesson — From Pattern to Formula (12 min)

On the board, write the table values in a column and have students help you identify the pattern algebraically:

Square 1: $1 = 2^0$

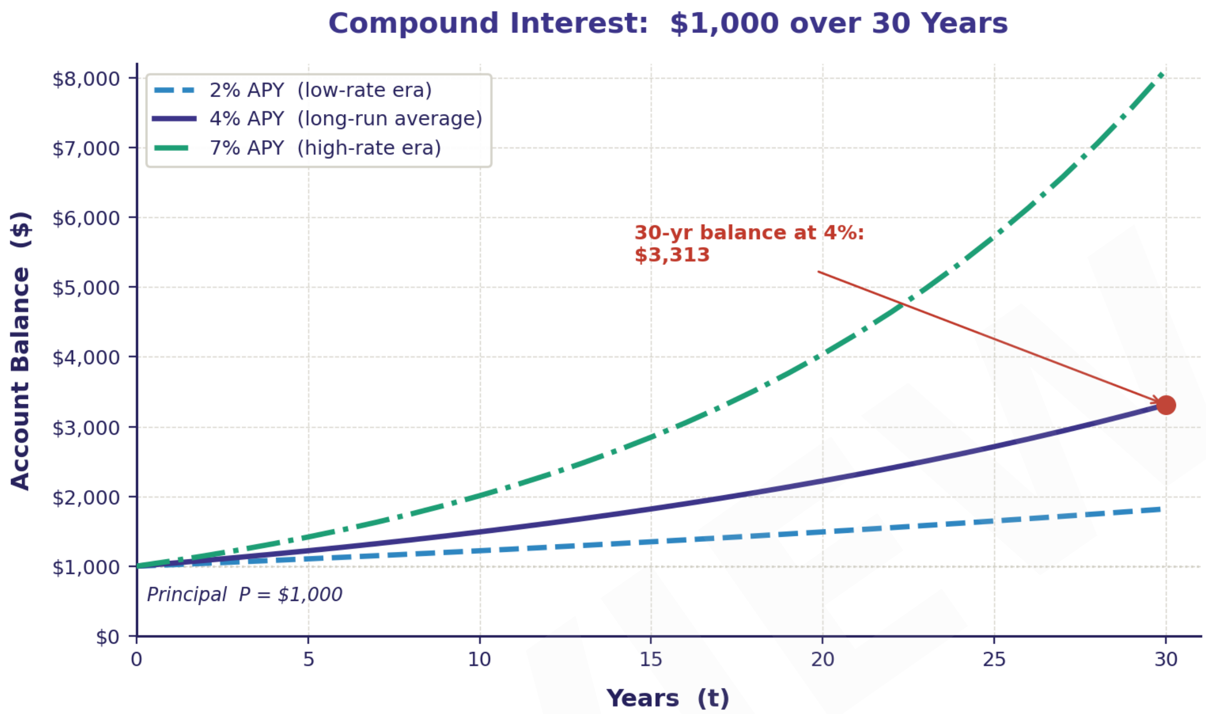

Figure 4. The same $1,000 principal grows very differently under different interest rates. At 4% APY (the long-run average), the balance reaches about $3,314 after 30 years — more than triple. At 7% APY (high-rate era), it reaches nearly $7,700.

Guided Practice (15 min)

Guided Practice — Compound Interest

Problem 1. Re-do the worked example using **annual** compounding ($n = 1$) instead of monthly. How much money is lost by compounding only once per year? (Round to the nearest dollar.)

Problem 2. A student deposits $2,500 at 6% APY compounded monthly. Find the balance after 10 years. Use the formula $A(t) = P \cdot (1 + r/n)^{nt}$.

Problem 3 — Compare. Two friends each invest $5,000. Friend A earns 5% compounded annually. Friend B earns 4.75% compounded monthly. Whose investment is worth more after 20 years? By how much? Use the frame: *"At t = 20, Friend A has ____ and Friend B has ____, so (A/B) has more by ____."*

Problem 4 — Predict. Using the long-run average rate of 4% compounded monthly, how long will it take for $1,000 to double? (Hint: set $A(t) = 2 \cdot 1000$ and solve for t. You may use the rule of 72 as a check: $72 \div 4 \approx 18$ years.)

Partner Discourse (5 min)

Use the **Compare-frame** stems to discuss the trade-off between rate and compounding frequency:

- *"A higher rate always produces a larger final balance because ____, but more frequent compounding also matters because ____."*

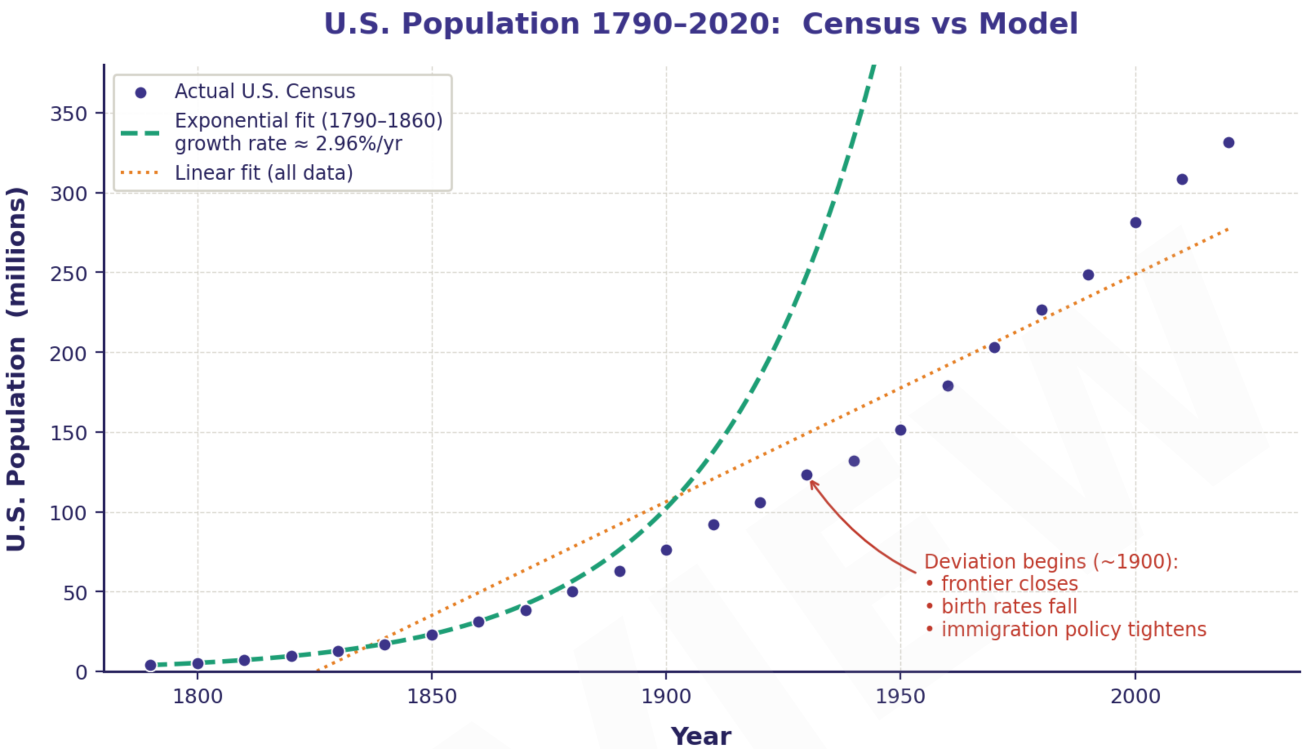

Figure 5. Actual U.S. census counts (purple dots) compared to a linear fit (orange dotted) and an exponential fit (teal dashed) derived from 1790–1860 data. The exponential model fits well until about 1900, then underestimates — population growth slowed as the frontier closed, birth rates fell, and immigration policy tightened.

Guided Practice (15 min)

Guided Practice — Modeling Census Data

Problem 1. Using the model $P(t) = 3.9 \cdot (1.0302)^t$, predict the U.S. population in 1900 ($t = 110$). The actual census count was 76.2 million. Compute the percent error of the model: (predicted − actual) / actual × 100%.

Problem 2. Predict the population in 2020 ($t = 230$) using the same model. Compare to the actual count of 331.4 million. By what factor does the model overestimate? Why does the model fail so badly over a long horizon?

Problem 3 — Compare. Fit a linear model to the 1790 and 1860 census points: $P(t) = 3.9 + m \cdot t$. Find the slope m. Then use the linear model to predict 1900 and 2020. Which model (linear or exponential) fits the 1790–1860 data better? Which fits the 1900–2020 data better? Use the frame: *"From 1790 to 1860 the (linear/exponential) model fits better because ____, but from 1900 to 2020 the (linear/exponential) model fits better because ____."*

Problem 4 — Predict. Using your preferred model from Problem 3, predict the U.S. population in 2050. State one assumption your prediction makes that may not hold.

Partner Discourse (5 min)

Lead a structured discussion about **why** the exponential model breaks down. Use the **Justify-frame**:

Worked Example — Reading the SIR-Lite Simulation

In a population of 1,000 people, 1 person is initially infected. The transmission rate is β = 0.35/day and the recovery rate is γ = 0.10/day (so the average infectious period is 10 days). Predict the basic reproduction number R_0, the peak number of infections, and the day of peak.

Step 1 — Compute R_0. $R_0 = \beta / \gamma = 0.35 / 0.10 = \mathbf{3.5}$.

Since $R_0 > 1$, the infection will spread.

Step 2 — Run the discrete simulation. Each day, update:

new infections = β · S · I / N
new recoveries = γ · I
S(t+1) = S(t) – new infections
I(t+1) = I(t) + new infections – new recoveries
R(t+1) = R(t) + new recoveries

Step 3 — Read the peak from Figure 7. The infection curve peaks on approximately Day 30 with about **350 people simultaneously infected**.

Step 4 — Compute the total infected. By Day 80, R ≈ 930 — meaning 93% of the population was eventually infected. Only about 70 people remained susceptible, a phenomenon called **herd immunity**.

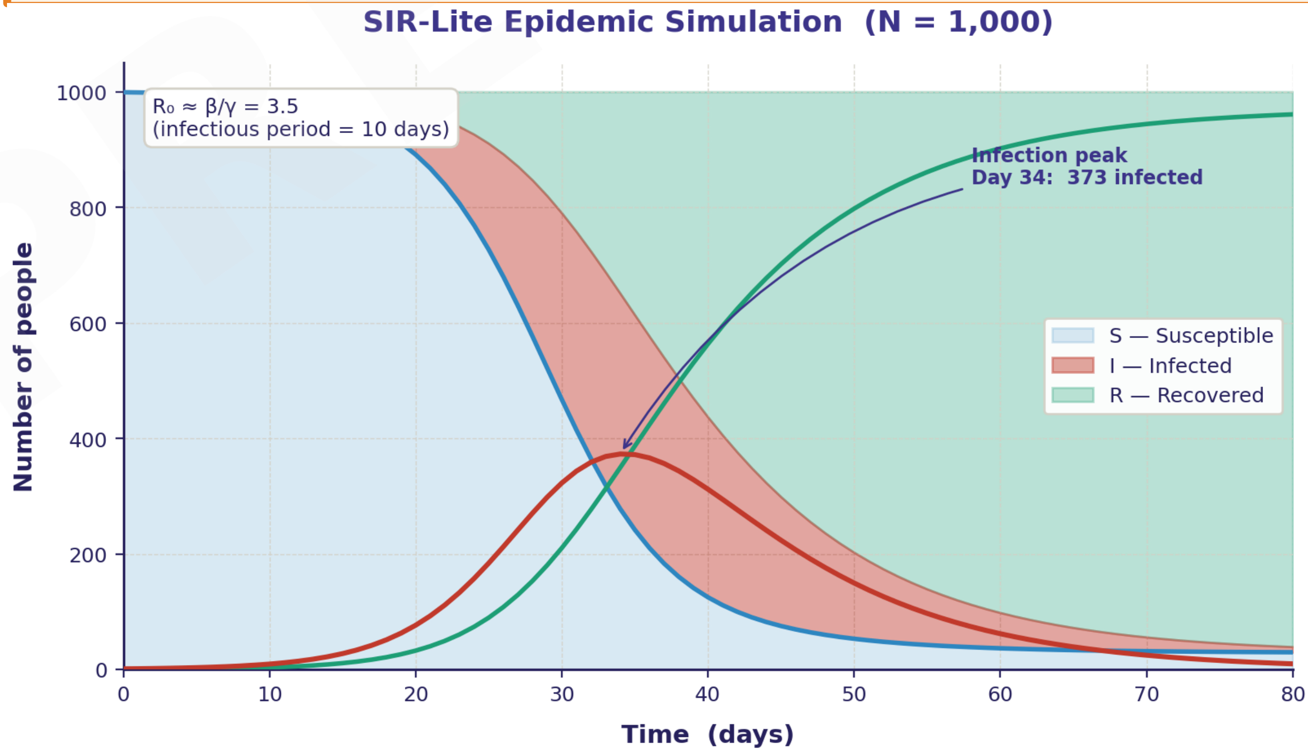

Figure 7. SIR-lite epidemic simulation for $R_0 \approx 3.5$ (β = 0.35, γ = 0.10). The blue area is the susceptible population, the red area is currently-infected, and the teal area is recovered. The infection peaks around Day 30 with about 350 simultaneous cases.

DAY 5 LESSON

Student Choice Board & Data Story Presentation

Lesson at a Glance

Time: 55 minutes (or split across 2 days: 1 build day + 1 presentation day)
Focus: Students choose one scenario, build a model, graph it, and present a "data story" to peers
Materials: Choice board (below), data-story template, peer-feedback forms, presentation rubric, graph paper or graphing tool
ELL Scaffolds Active: Data-story template with sentence starters; partner rehearsal before whole-class presentation; peer-feedback stems

The Choice Board

Students choose **one** of the four scenarios below to investigate independently. Each option has the same three deliverables — a model, a labeled graph, and a data story — but the scenario and dataset differ. Encourage students to choose the option that most interests them: research on self-determination theory (Deci & Ryan, 2000) shows that choice increases both engagement and persistence.

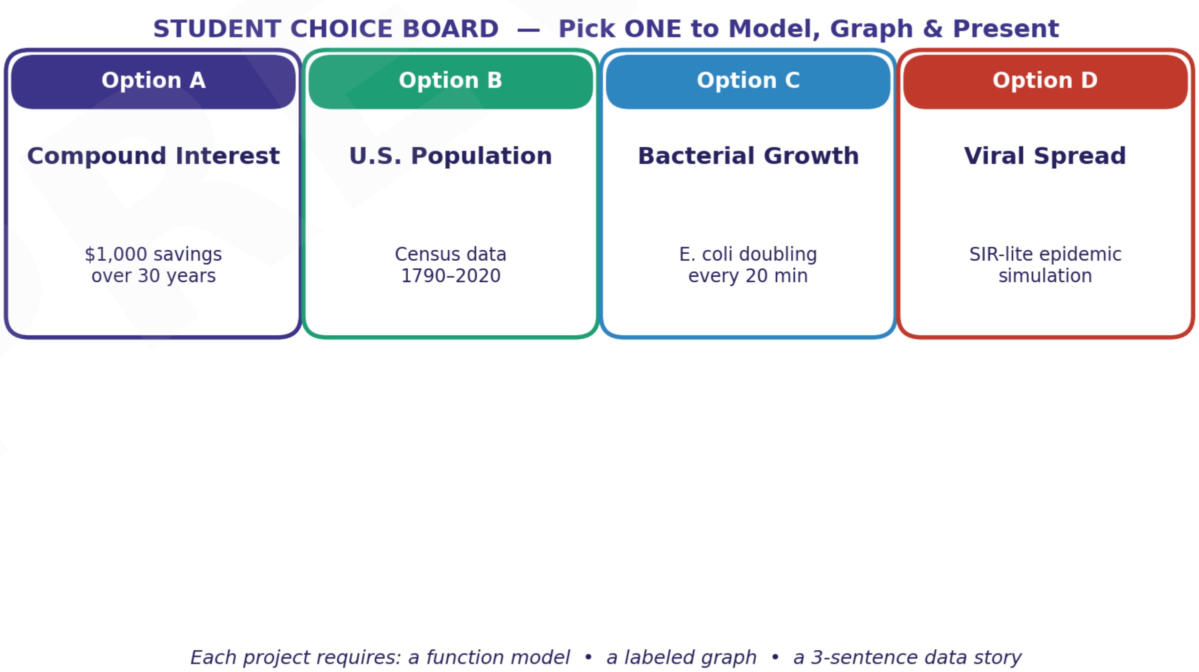

Figure 8. The four scenarios students may choose from. Each project requires the same three deliverables: a model, a graph, and a 3-sentence data story.

Project Requirements (All Options)

ANSWER KEY

Detailed Solutions & Worked Examples

Every problem in this unit is solved below with the full reasoning shown. Each solution follows the same four-step structure: ***Given → Formula → Substitution → Answer with interpretation****. Teachers may distribute these solutions to students as a self-study reference, or use them as a grading key. ELL students benefit from seeing the verbal interpretation step modeled explicitly — it shows how mathematical results are translated back into real-world meaning.*

Day 1 — Patterns That Multiply

Guided Practice 1 — $500 at 3% per year

Given: P = \$500, growth rate r = 3% = 0.03, time t = 10 years

Formula: $A(t) = P \cdot (1 + r)^t = 500 \cdot (1.03)^t$

Substitute: $A(10) = 500 \cdot (1.03)^{10}$

Compute: $(1.03)^{10} \approx 1.3439$, so $A(10) \approx 500 \cdot 1.3439 \approx 671.96$

Answer: A(10) ≈ \$671.96. The account grows by a factor of 1.3439 over 10 years, meaning the original \$500 has earned about \$172 in interest. Notice that "3% per year" does *not* mean "30% over 10 years" — the multiplicative structure produces a larger result (\$171.96 vs. \$150) because each year's interest earns interest in subsequent years.

Guided Practice 2 — Fish population tripling every 5 years

Given: P_0 = 1,200 fish, tripling every 5 years. Time measured in 5-year intervals: t = 3 means 15 years.

Formula: $P(t) = 1{,}200 \cdot 3^t$, where t is the number of 5-year intervals.

Substitute: $P(3) = 1{,}200 \cdot 3^3 = 1{,}200 \cdot 27$

Compute: P(3) = 32,400 fish

Answer: P(15 years) = 32,400 fish. If instead you wanted to measure t in years directly, the per-year growth factor would be $3^{1/5} \approx 1.2457$, giving $P(t) = 1{,}200 \cdot (1.2457)^t$. Both models produce the same answer at t = 15 years.

ASSESSMENT

Rubrics — Project & Participation

*Two rubrics are provided. The **Project Rubric** scores the culminating Day 5 data-story presentation across four criteria on a 4-point scale. The **Participation & Discourse Rubric** provides daily formative feedback on the language and collaboration skills that this unit deliberately develops. Both rubrics include ELL-specific criteria so that language growth is rewarded alongside mathematical growth.*

Project Rubric — Day 5 Data Story

Score each criterion 1–4. Total possible: 16 points. Convert to a percentage using the standard 90/80/70/60 scale.

Criterion	4 — Exemplary	3 — Proficient	2 — Developing	1 — Emerging
Mathematical Model ($A(t) = A_0 \cdot b^t$)	**4 — Exemplary.** Model is correctly written with units; A_0 and b are accurately derived from real data; student explains why this model form is appropriate.	**3 — Proficient.** Model is correct with units; A_0 and b are accurate; explanation is brief or generic.	**2 — Developing.** Model has a minor error (wrong units, incorrect b) OR explanation is missing; the form $A_0 \cdot b^t$ is used.	**1 — Emerging.** Model is missing, incorrect in form, or shows no connection to the chosen scenario.
Labeled Graph	**4 — Exemplary.** Both axes labeled with units; y-intercept marked; ≥3 key points identified (e.g., doubling points); caption explains what the graph shows; curve is smooth and accurate.	**3 — Proficient.** Axes labeled; y-intercept marked; ≥2 key points identified; caption present.	**2 — Developing.** Axes labeled but missing units; OR only y-intercept marked; OR caption missing or generic.	**1 — Emerging.** Graph is missing, unlabeled, or fundamentally misrepresents the model.
Data Story (3–5 sentences)	**4 — Exemplary.** All 7 template sections addressed; ≥2 academic vocabulary terms used correctly; limitation and real-world implication are specific and insightful.	**3 — Proficient.** 5–6 sections addressed; ≥1 vocabulary term used; limitation and implication present but general.	**2 — Developing.** 3–4 sections addressed; vocabulary terms absent or used incorrectly; either limitation or implication missing.	**1 — Emerging.** Fewer than 3 sections addressed; no academic vocabulary; limitation and implication absent.
Oral Presentation (2–3 min)	**4 — Exemplary.** Uses ≥3 sentence starters from the template; speaks audibly; makes eye contact; answers peer question with a Justify-frame stem; refers to graph during explanation.	**3 — Proficient.** Uses 2 sentence starters; mostly audible; answers peer question with some justification.	**2 — Developing.** Uses 1 sentence starter; voice too soft OR no eye contact; peer question answered briefly without justification.	**1 — Emerging.** No sentence starters used; presentation is under 1 minute; cannot answer peer question.

Daily Participation & Discourse Rubric

Use this rubric to give quick formative feedback during Days 1–4. It can also be used by students to self-assess at the end of each lesson. The criteria are deliberately ELL-inclusive: they reward **effort to use academic English**

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of complete instructional content

5

DAYS

of ready-to-teach PBL lessons

4

DATASETS

real federal rates, census, lab, SIR model

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