CONIC SECTIONS

Complete Unit

Circles • Ellipses • Parabolas • Hyperbolas

Grades 10–11

Includes: Guided Notes, Discovery Activities, Worked Examples with Accurate Graphs, Practice Sets, Real-World Applications, Quizzes, and a Comprehensive Unit Test.

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TOPIC 1: CIRCLES

Standard Form of a Circle

The standard equation of a circle with center **(h, k)** and radius **r** is:

$$(x - h)^2 + (y - k)^2 = r^2$$

• If the center is at the origin (0,0), the equation simplifies to: $x^2 + y^2 = r^2$

• To find the center and radius from standard form, identify h, k, and r. Remember that the signs inside the parentheses are flipped from the coordinate!

General Form of a Circle

When expanded, the standard form becomes the general form:

$$Ax^2 + Ay^2 + Dx + Ey + F = 0$$

(Note: The coefficients of x^2 and y^2 must be the same and non-zero for a circle).

Converting General Form to Standard Form

To graph a circle from general form, you must complete the square for both x and y.

Steps to Complete the Square:

1. Group x-terms and y-terms together, moving the constant to the right side.
2. Factor out the leading coefficient from both groups (if it's not 1).
3. Take half of the x-coefficient, square it, and add it to BOTH sides. Repeat for y.
4. Factor the perfect square trinomials on the left, and simplify the right side.

DISCOVERY ACTIVITY: Deriving the Circle Equation

Objective: Use the Distance Formula to derive the equation of a circle.

Background: A circle is defined as the set of all points in a plane that are equidistant from a fixed point called the center. The fixed distance is the radius, **r**.

Step 1: Let the center of a circle be at point C(h, k). Let (x, y) be ANY point on the circle. Write the distance formula between (x, y) and (h, k):

$$d = \sqrt{(x - h)^2 + (y - k)^2}$$

Step 2: Because (x, y) is on the circle, the distance d must equal the radius, r. Substitute r for d:

$$r = \sqrt{(x - h)^2 + (y - k)^2}$$

Step 3: Square both sides to eliminate the square root.

$$r^2 = (x - h)^2 + (y - k)^2$$

➤ **Conclusion:** You have just derived the standard equation of a circle!

Worked Examples: Circles

Example 1: Graphing from Standard Form

Problem: Identify the center and radius of the circle with equation $(x - 2)^2 + (y + 1)^2 = 9$. Graph the circle and label the center and radius.

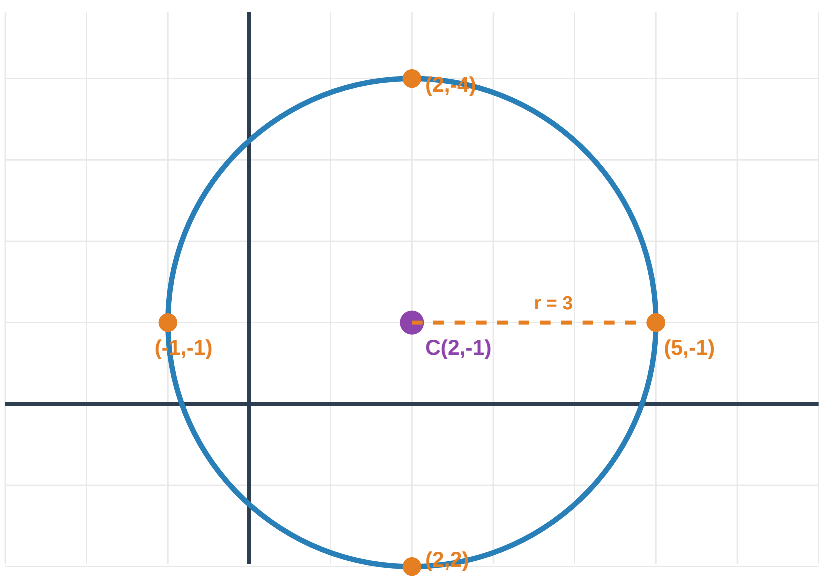
Solution:

Step 1: Compare to standard form $(x - h)^2 + (y - k)^2 = r^2$.

- x-term: $(x - 2)^2$ means $h = 2$
- y-term: $(y + 1)^2 = (y - (-1))^2$ means $k = -1$
- Constant: $r^2 = 9$ means $r = 3$

Step 2: Identify key features. Center = $(2, -1)$, Radius = 3

Step 3: Graph by plotting the center, then counting 3 units up, down, left, and right to find 4 points on the circle. Draw the curve.



Example 2: Converting General Form to Standard Form

Problem: Write the equation $\mathbf{x^2 + y^2 - 6x + 4y - 12 = 0}$ in standard form. Identify the center and radius, and graph the circle.

Solution:

Step 1: Group x-terms and y-terms. Move the constant to the right side.

$$(x^2 - 6x) + (y^2 + 4y) = 12$$

Step 2: Complete the square for both groups.

• For x: Half of –6 is –3. $(-3)^2 = 9$. Add 9 to both sides.

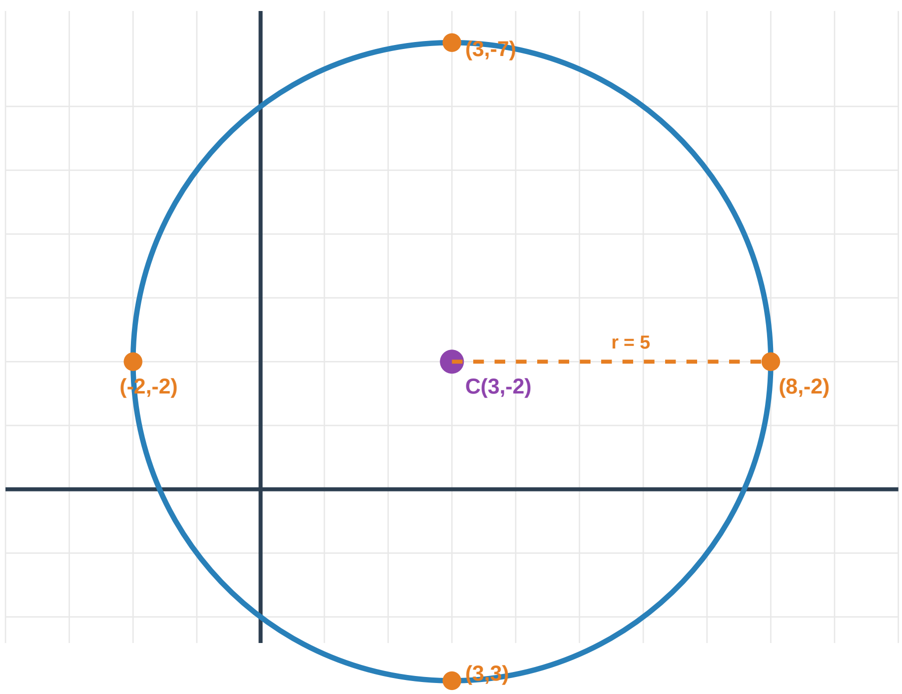
• For y: Half of 4 is 2. $(2)^2 = 4$. Add 4 to both sides.

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$$

Step 3: Factor the perfect square trinomials and simplify the right side.

$$\mathbf{(x - 3)^2 + (y + 2)^2 = 25}$$

Step 4: Identify key features. Center = (3, –2), $r^2 = 25 \rightarrow r = 5$



TOPIC 2: ELLIPSES

Standard Form of an Ellipse

An ellipse is the set of all points (x, y) such that the sum of the distances from (x, y) to two fixed points (foci) is constant.

Horizontal Major Axis:

$$(x - h)^2 / a^2 + (y - k)^2 / b^2 = 1 \quad \text{(where } a > b > 0\text{)}$$

- Center: (h, k)
- Vertices: $(h \pm a, k)$
- Co-vertices: $(h, k \pm b)$
- Foci: $(h \pm c, k)$ where $c^2 = a^2 - b^2$

Vertical Major Axis:

$$(x - h)^2 / b^2 + (y - k)^2 / a^2 = 1 \quad \text{(where } a > b > 0\text{)}$$

- Center: (h, k)
- Vertices: $(h, k \pm a)$
- Co-vertices: $(h \pm b, k)$
- Foci: $(h, k \pm c)$ where $c^2 = a^2 - b^2$

Eccentricity: Measures how 'stretched' an ellipse is. $e = c / a$. $(0 < e < 1)$. As e approaches 1, the ellipse becomes more elongated.

DISCOVERY ACTIVITY: Deriving the Ellipse Equation

Objective: Use the definition of an ellipse to derive its standard equation.

Setup: Place two pushpins (foci) on the x-axis at (-c, 0) and (c, 0). Loop a string of length 2a around the pins (where $2a > 2c$). Pull the string taut with a pencil and trace the curve.

Step 1: Let (x, y) be any point on the ellipse. The distance from (x, y) to $F_1(-c, 0)$ is $d_1 = \sqrt{(x + c)^2 + y^2}$.

Step 2: The distance from (x, y) to $F_2(c, 0)$ is $d_2 = \sqrt{(x - c)^2 + y^2}$.

Step 3: By definition, $d_1 + d_2 = 2a$. Substitute the distance formulas:

$$\sqrt{(x + c)^2 + y^2} + \sqrt{(x - c)^2 + y^2} = 2a$$

Step 4: Isolate one radical and square both sides. Then isolate the remaining radical and square both sides again.

After algebraic simplification (expanding and canceling $2x^2$ and $2y^2$ terms), you get: $a^2 - cx = a\sqrt{(x - c)^2 + y^2}$

Step 5: Square both sides one last time, expand, and cancel. Group x^2 and c^2 terms to find:

$$a^2x^2 + a^2c^2 - c^2x^2 = a^2(a^2 - c^2) \rightarrow (a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

Step 6: Since $a > c$, let $b^2 = a^2 - c^2$. Substitute b^2 into the equation:

$$b^2x^2 + a^2y^2 = a^2b^2 \rightarrow \text{Divide both sides by } a^2b^2 \rightarrow x^2/a^2 + y^2/b^2 = 1$$

Worked Examples: Ellipses

Example 1: Graphing an Ellipse (Horizontal Major Axis)

Problem: Identify the center, vertices, co-vertices, foci, and eccentricity of the ellipse $\mathbf{(x + 1)^2 / 25 + (y - 2)^2 / 9 = 1}$. Graph the ellipse.

Solution:

Step 1: Identify h, k, a^2, and b^2.

• $(x - (-1))^2 / 25 + (y - 2)^2 / 9 = 1 \rightarrow h = -1, k = 2$

• $a^2 = 25 \rightarrow a = 5$. $b^2 = 9 \rightarrow b = 3$.

• Since the larger denominator (25) is under the x-term, the major axis is horizontal.

Step 2: Find c (distance from center to foci).

• $c^2 = a^2 - b^2 = 25 - 9 = 16 \rightarrow c = 4$.

Step 3: Calculate key features.

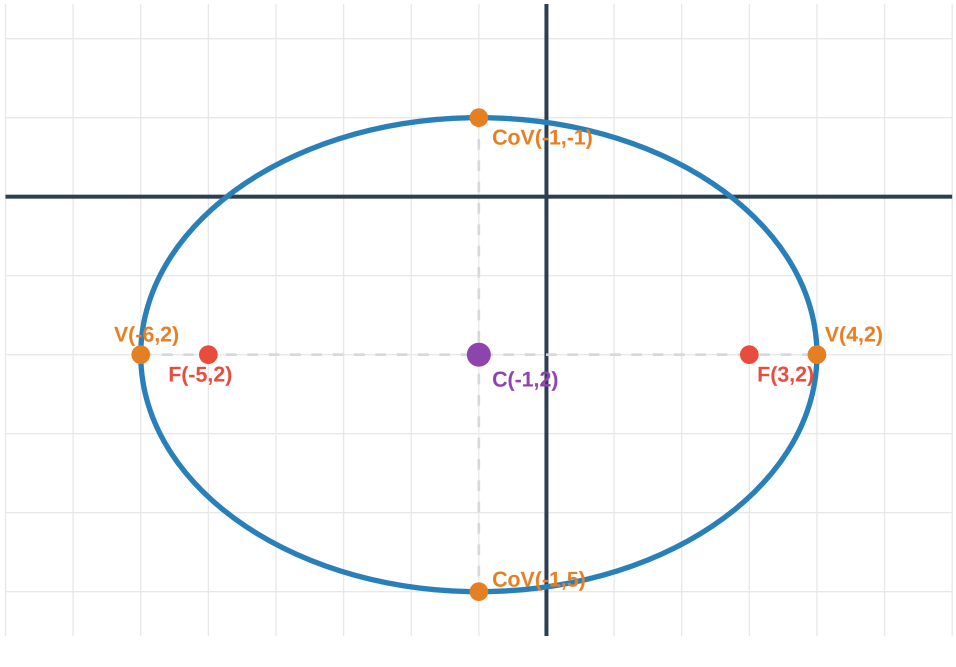
• Center: (-1, 2)

• Vertices: $(h \pm a, k) \rightarrow (-1 \pm 5, 2) \rightarrow$ **(4, 2) and (-6, 2)**

• Co-vertices: $(h, k \pm b) \rightarrow (-1, 2 \pm 3) \rightarrow$ **(-1, 5) and (-1, -1)**

• Foci: $(h \pm c, k) \rightarrow (-1 \pm 4, 2) \rightarrow$ **(3, 2) and (-5, 2)**

• Eccentricity: $e = c/a = 4/5 =$ **0.8**


CoV(-1,-1)
V(-6,2)
F(-5,2)
C(-1,2)
F(3,2)
V(4,2)
CoV(-1,5)

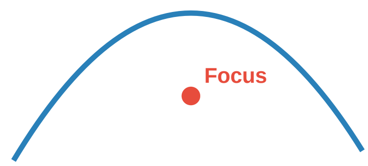
REAL-WORLD APPLICATIONS

1. Satellite Dish

A satellite dish is 12 feet in diameter and 2 feet deep. Where should the receiver be placed?

Solution:

1. Model: $x^2 = 4py$, vertex at origin.
2. Edge at x=6, y=2: $36 = 8p \rightarrow p = 4.5$.
3. Place receiver at focus: **4.5 ft from vertex**.



2. Suspension Bridge

A suspension bridge cable hangs as a parabola. Towers are 100 m apart and 20 m tall. The lowest point touches the deck. How high is the cable 25 m from center?

Solution:

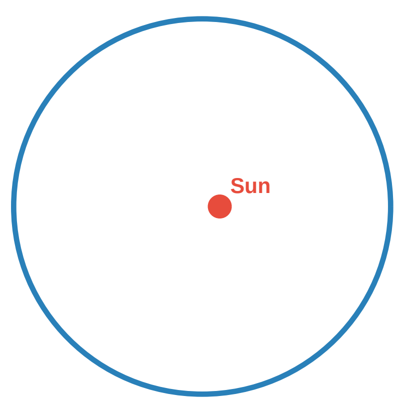
1. Model: $x^2 = 4py$, vertex at (0,0). Tower at x=50, y=20.
2. $2500 = 80p \rightarrow p = 31.25$. Equation: $x^2 = 125y$.
3. At x=25: $625 = 125y \rightarrow$ **y = 5 meters**.

3. Planetary Orbit

Mars orbits the Sun in an ellipse (a=228M km, e=0.0934). Find perihelion (closest distance).

Solution:

1. $c = 228 \times 0.0934 = 21.295$ M km.
2. Perihelion = a – c = **206.705 million km**.



4–20. Additional Real-World Applications

COMPREHENSIVE UNIT TEST

1. Write the standard form equation of a circle with center (-4, 0) and radius $\sqrt{7}$.

2. Convert $\mathbf{x^2 + y^2 + 6x - 8y - 11 = 0}$ to standard form. State the center and radius.

3. Graph the ellipse $\mathbf{x^2/25 + y^2/9 = 1}$. Label the center, vertices, and foci.

4. Convert $\mathbf{9x^2 + 4y^2 - 18x + 16y - 11 = 0}$ to standard form. Identify all key features.

5. Identify the vertex, focus, and directrix of $\mathbf{x^2 = -16y}$.

6. Convert $\mathbf{x^2 - 6x + 2y + 11 = 0}$ to standard form. Identify the vertex and direction of opening.

7. Graph the hyperbola $\mathbf{y^2/16 - x^2/4 = 1}$. Label the center, vertices, foci, and asymptotes.

8. Convert $\mathbf{4x^2 - 9y^2 + 24x + 18y - 9 = 0}$ to standard form. Identify the center and vertices.

9. Identify the conic section using the discriminant: $\mathbf{5x^2 + 5y^2 - 10x + 20y - 5 = 0}$.

10. A flashlight reflector is 8 inches wide and 3 inches deep. How far from the vertex should the light bulb be placed?

Thank You!

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