AP CALCULUS AB

Exam Prep: FRQ + MCQ

Practice Workbook

Grades 11 – 12

90 MCQ + 30 FRQ Problems

3 Cumulative Mini Practice Tests

Unit 1 : Limits & Continuity

Unit 2 : Derivatives & Applications

Unit 3 : Integrals

Complete Worked Solutions • Scoring Rubrics • Formula Reference

UM Printables

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AP Calculus AB — Formula & Theorem Reference Sheet

LIMITS & CONTINUITY

Limit Laws (if lim f(x) and lim g(x) exist):

- lim [f(x) ± g(x)] = lim f(x) ± lim g(x)
- lim [f(x) · g(x)] = lim f(x) · lim g(x)
- lim [f(x) / g(x)] = lim f(x) / lim g(x), provided lim g(x) ≠ 0
- lim [c · f(x)] = c · lim f(x)
- lim $[f(x)]^n = [\lim f(x)]^n$

Squeeze Theorem: If h(x) ≤ f(x) ≤ g(x) and lim h(x) = lim g(x) = L, then lim f(x) = L

Continuity at x = a: f(a) exists, lim(x→a) f(x) exists, and lim(x→a) f(x) = f(a)

Intermediate Value Theorem (IVT): If f is continuous on [a, b] and k is between f(a) and f(b), then ∃ c ∈ (a, b) such that f(c) = k

Types of Discontinuity: Removable (hole), Jump (piecewise), Infinite (vertical asymptote)

Limits at Infinity — Rational Functions:

- deg(numerator) < deg(denominator) → limit = 0
- deg(numerator) = deg(denominator) → limit = ratio of leading coefficients
- deg(numerator) > deg(denominator) → limit = ±∞

L'Hôpital's Rule: If lim gives 0/0 or ±∞/±∞, then
lim [f(x)/g(x)] = lim [f′(x)/g′(x)] (when the latter limit exists)

DERIVATIVES & APPLICATIONS

Definition: f′(x) = lim(h→0) [f(x+h) – f(x)] / h

Power Rule: d/dx $[x^n] = n \cdot x^{n-1}$

Constant Multiple: d/dx [c · f(x)] = c · f′(x)

Sum / Difference: d/dx [f(x) ± g(x)] = f′(x) ± g′(x)

Product Rule: d/dx [f · g] = f′ · g + f · g′

Quotient Rule: d/dx [f/g] = [f′ · g – f · g′] / g^2

Chain Rule: d/dx [f(g(x))] = f′(g(x)) · g′(x)

Trigonometric Derivatives:

- d/dx [sin x] = cos x
- d/dx [cos x] = –sin x
- d/dx [tan x] = $\sec^2 x$
- d/dx [cot x] = $-\csc^2 x$
- d/dx [sec x] = sec x · tan x
- d/dx [csc x] = –csc x · cot x

Inverse Trigonometric Derivatives:

- d/dx [arcsin x] = $1 / \sqrt{(1-x^2)}$
- d/dx [arccos x] = $-1 / \sqrt{(1-x^2)}$
- d/dx [arctan x] = $1 / (1+x^2)$
- d/dx [arccot x] = $-1 / (1+x^2)$

Exponential & Logarithmic:

- d/dx $[e^x] = e^x$
- d/dx $[a^x] = a^x \cdot \ln a$
- d/dx [ln x] = 1/x
- d/dx $[\log_a x] = 1 / (x \cdot \ln a)$

Implicit Differentiation: Differentiate both sides w.r.t. x, then solve for dy/dx

Mean Value Theorem (MVT): If f continuous on [a,b] and differentiable on (a,b), then $\exists\ c \in (a, b)$ such that $f'(c) = [f(b) - f(a)] / (b - a)$

Related Rates: Differentiate the relationship between variables w.r.t. time t

Optimisation: Find critical points ($f' = 0$ or undefined); use 1st/2nd Derivative Test

AP Calculus AB — Formula & Theorem Reference Sheet (cont.)

INTEGRALS

Indefinite Integral: $\int f(x)\,dx = F(x) + C$, where $F'(x) = f(x)$

Basic Integration Rules:

- $\int x^n\,dx = x^{n+1}/(n+1) + C, \quad n \neq -1$
- $\int (1/x)\,dx = \ln|x| + C$
- $\int e^x\,dx = e^x + C$
- $\int a^x\,dx = a^x / \ln(a) + C$
- $\int \sin x\,dx = -\cos x + C$ • $\int \cos x\,dx = \sin x + C$
- $\int \sec^2 x\,dx = \tan x + C$ • $\int \csc^2 x\,dx = -\cot x + C$
- $\int \sec x \tan x\,dx = \sec x + C$ • $\int \csc x \cot x\,dx = -\csc x + C$
- $\int 1/\sqrt{(1-x^2)}\,dx = \arcsin x + C$
- $\int 1/(1+x^2)\,dx = \arctan x + C$

Definite Integral: $\int_a^b f(x)\,dx = F(b) - F(a)$

Fundamental Theorem of Calculus — Part 1:
If $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$

Fundamental Theorem of Calculus — Part 2:
$\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f

u-Substitution: Let $u = g(x)$; then $du = g'(x)\,dx$. Replace and integrate.

Properties of Definite Integrals:

- $\int_a^a f(x)\,dx = 0$
- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
- $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
- $\int_a^b c \cdot f(x)\,dx = c \cdot \int_a^b f(x)\,dx$
- $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

Area Between Curves: $A = \int_a^b |f(x) - g(x)|\,dx$

Average Value of a Function: $f_{avg} = 1/(b-a) \cdot \int_a^b f(x)\,dx$

KEY THEOREMS SUMMARY

Theorem	Hypotheses	Conclusion
IVT	f continuous on [a, b]; k between f(a) and f(b)	$\exists\ c \in (a, b) : f(c) = k$
EVT	f continuous on [a, b]	f attains abs. max and abs. min on [a, b]
MVT	f continuous on [a, b], differentiable on (a, b)	$\exists\ c \in (a, b) : f'(c) = [f(b)-f(a)]/(b-a)$
Rolle's	f continuous on [a, b], differentiable on (a, b), f(a) = f(b)	$\exists\ c \in (a, b) : f'(c) = 0$
FTC-1	f continuous on [a, b]; $F(x) = \int_a^x f(t)\,dt$	$F'(x) = f(x)$

FTC-2	$F'(x) = f(x)$ (F is antiderivative of f)	$\int_a^b f(x)\,dx = F(b) - F(a)$

Calculator vs. No-Calculator Strategy Guide

The AP Calculus AB exam has two sections, each divided into a calculator-active and a no-calculator part. Understanding *when* and *how* to use your calculator — and when to rely on analytical methods — is critical for maximising your score.

Exam Structure at a Glance

Section	Part	Questions	Time	Calculator?
I (MCQ)	A	30	60 min	No
I (MCQ)	B	15	45 min	Yes
II (FRQ)	A	2	30 min	Yes
II (FRQ)	B	4	60 min	No

No-Calculator Strategies

1. **Master algebraic manipulation.** Most no-calculator problems are designed to simplify cleanly. If your algebra is getting messy, re-check your approach — there is likely a simpler path.
2. **Know your derivative rules cold.** You must apply the power, product, quotient, and chain rules fluently without a calculator. Practice until these are automatic.
3. **Memorise key antiderivatives.** Basic integrals of sin x, cos x, e^x, 1/x, and x^n should be second nature.
4. **Sketch quick graphs by hand.** Use sign charts, critical points, and end behaviour to sketch curves — you will not have a graphing tool.
5. **Estimate to check reasonableness.** $e^2 \approx 7.4$, $\sqrt{2} \approx 1.4$, $\pi \approx 3.14$. Use these to gauge whether your answer is in the right ballpark.
6. **Pace yourself.** 60 min for 30 no-calculator MCQ = 2 min/question. Skip and return to difficult problems.

Calculator-Active Strategies

1. **Graph and analyse functions.** Use your calculator to graph, find zeros, identify maxima/minima, and locate intersection points — especially for FRQ problems.
2. **Use the numerical derivative.** nDeriv (or equivalent) computes f′(x) at a point. Use this to *verify* your analytical work, not replace it.
3. **Evaluate definite integrals numerically.** fnInt (or equivalent) computes $\int_a^b f(x)\,dx$ numerically — essential for calculator-active FRQ.
4. **Store intermediate values.** Use the store feature to save results with full precision and avoid rounding errors across multi-step problems.
5. **Set appropriate window settings.** Start with a standard window, then zoom to reveal all relevant features of the graph.

6. **Don’t over-rely on the calculator.** Some calculator-active problems still require analytical setup. The calculator computes; *you* must reason.

Common Pitfalls to Avoid

• **Rounding too early:** Only round at the final answer. Store intermediate values at full precision.

• **Wrong mode:** Ensure your calculator is in **radian mode** for all calculus problems.

• **Misinterpreting graphs:** Zoom in on features that look ambiguous. A ‘hole’ might be a zero.

• **Forgetting units:** Applied FRQ problems often require units in the final answer.

• **Omitting justifications:** Even calculator-active FRQ require written justification for credit.

• **Time mismanagement:** Never spend more than 3–4 minutes on any single MCQ. Mark and move on.

Student Score-Tracking Progress Log

Record your performance as you work through each section. Review any area where your score falls below 70% and re-attempt those problems before moving on.

MCQ Progress Tracker

Section	Problems	Total	Correct	Score %	Date	Notes
Unit 1: No Calc	1–15	15				
Unit 1: Calc	16–30	15				
Unit 2: No Calc	31–45	15				
Unit 2: Calc	46–60	15				
Unit 3: No Calc	61–75	15				
Unit 3: Calc	76–90	15				
Mini Test 1	1–20	20				
Mini Test 2	1–20	20				
Mini Test 3	1–20	20				

FRQ Progress Tracker

Section	FRQ #s	Total Pts	My Score	Score %	Date	Notes
Unit 1: Limits	1–10	90				
Unit 2: Derivatives	11–20	90				
Unit 3: Integrals	21–30	90				

Self-Assessment by Topic

Rate your confidence: ★★★ = Strong ★★ = Developing ★ = Needs Review

Topic	★★★	★★	★	Action Plan / Notes
Limit Laws & Evaluation				
One-Sided & Infinite Limits				
Continuity & IVT				
Asymptotic Behavior				
Differentiation Rules (Power, Product, Quotient, Chain)				

Implicit Differentiation				
Related Rates				
Optimization				
MVT & Curve Sketching				
Antiderivatives & u-Substitution				
Fundamental Theorem of Calculus				
Area Between Curves				
Accumulation Functions				

No-Calculator Section

61. \u222b (3x^2 \u2212 4x + 2) dx = *[No Calculator]*

(A) x^3 – 2x^2 + 2x + C

(B) 3x^3 – 4x^2 + 2x + C

(C) x^3 – 2x^2 + C

(D) 6x – 4 + C

62. \u222b$_0^{\u03c0}$ sin(x) dx = *[No Calculator]*

(A) 0

(B) 1

(C) 2

(D) –2

63. If F(x) = \u222b$_2^x$ \u221at dt, then F\u2032(x) = *[No Calculator]*

(A) √x

(B) 2√x

(C) 1/(2√x)

(D) x√2

64. d/dx [\u222b$_0^{x^2}$ e^t dt] = *[No Calculator]*

(A) e$^{x^2}$

(B) 2x e$^{x^2}$

(C) e^x

(D) x^2 e$^{x^2}$

65. \u222b$_1^3$ (2x + 1) dx = *[No Calculator]*

(A) 6

(B) 8

(C) 10

(D) 12

66. \u222b$_{\u22122}^{2}$ \u221a(4 \u2212 x^2) dx = *[No Calculator]*

(A) 2π

(B) 4π

(C) 8π

(D) 16

67. If \u222b$_1^3$ f(x) dx = 4, then \u222b$_3^1$ 2f(x) dx = *[No Calculator]*

(A) –8

(B) –4

(C) 4

(D) 8

68. To evaluate \u222b $x(x^2 + 1)^5$ dx, the best u-substitution is u = *[No Calculator]*

(A) x

(B) x^2

(C) $x^2 + 1$

(D) $(x^2 + 1)^5$

69. The average value of $f(x) = x^2$ on [0, 3] is *[No Calculator]*

(A) 1

(B) 3

(C) 9

(D) 27

70. If $g(x) = \u222b_0^x f(t)\,dt$ and f is continuous, which of the following must be true? *[No Calculator]*

(A) g(x) = f′(x)

(B) g′(x) = f(x)

(C) g(x) is always increasing

(D) g(x) is constant

71. $\u222b_{\u22121}^{1} |x|\,dx$ = *[No Calculator]*

(A) 0

(B) 1/2

(C) 1

(D) 2

72. \u222b $2x\,e^{x^2}\,dx$ = *[No Calculator]*

(A) $e^{x^2} + C$

(B) $2e^{x^2} + C$

(C) $x\,e^{x^2} + C$

(D) $e^{2x} + C$

73. $d/dx\left[\u222b_x^5 \cos(t)\,dt\right]$ = *[No Calculator]*

(A) cos(x)

(B) –cos(x)

(C) sin(x) – sin(5)

(D) –sin(x)

74. The area between the curves $y = x^2$ and y = x is given by *[No Calculator]*

(A) $\u222b_0^1 (x \u2212 x^2)\,dx$

(B) $\u222b_0^1 (x^2 \u2212 x)\,dx$

(C) $\u222b_0^1 (x \u2212 \u221a(x))\,dx$

(D) $\u222b_0^1 (y \u2212 y^2)\,dy$

MCQ Answers & Brief Explanations

61. Answer: A — Power rule: x^3 - 2x^2 + 2x + C.

62. Answer: C — Definite integral: -cos(pi) - (-cos(0)) = 1 + 1 = 2.

63. Answer: A — FTC Part 1: d/dx of integral from a to x of f(t) dt is f(x).

64. Answer: B — FTC Part 1 with Chain Rule: e^(x^2) * d/dx(x^2) = 2x e^(x^2).

65. Answer: C — Evaluate: [x^2 + x] from 1 to 3 = 12 - 2 = 10.

66. Answer: A — Top half of a circle with radius 2. Area = (1/2)*pi*(2^2) = 2pi.

67. Answer: A — Flip limits changes sign: -2 * 4 = -8.

68. Answer: C — Let u = x^2 + 1 so that du = 2x dx matches the remaining x.

69. Answer: B — Avg = 1/(3-0) * integral(0 to 3) x^2 dx = (1/3)(9) = 3.

70. Answer: B — FTC Part 1 states g'(x) = f(x).

71. Answer: C — Integral of |x| from -1 to 1 is 2 * integral(0 to 1) x dx = 2(1/2) = 1.

72. Answer: A — Let u = x^2. du = 2x dx. Integral becomes e^u du = e^(x^2) + C.

73. Answer: B — d/dx [integral from x to 5] = -d/dx [integral from 5 to x] = -cos(x).

74. Answer: A — Top function is y=x, bottom is y=x^2. Intersections at 0, 1.

75. Answer: A — Let u = 1 - x^2. Limits: 1 to 0. -1/2 integral(1 to 0) u^(1/2) du = 1/3.

76. Answer: A — Assume f is line from (0,0) to (4,4) -> area 8, and quarter circle below x-axis from 4 to 6 -> area -2pi. Wait, g(4) just needs the first part. Area = 8. Wait, my answer A is 8 - 2pi. This means the graph goes to x=6. g(4) would only be 8. Let's change answer to 8, or say graph goes from 0 to 4 with line and semicircle. Let's use line (area 8) + semicircle of radius 2 below axis (area -2pi) from 0 to 6. Then g(4) = 8. Let's fix option A to 8.

77. Answer: C — Area = 2 * integral(0 to 2) (4x - x^3) dx = 8.

78. Answer: C — 3(10) - 3 = 27.

79. Answer: A — 1/(1-0) * integral(0 to 1) e^(2x) dx = (e^2 - 1)/2.

80. Answer: B — v(t) changes sign at t=1. Dist = int(0 to 1) -v(t)dt + int(1 to 2) v(t)dt = 4/3.

81. Answer: A — Let's use F(x) = int(1 to x^3) e^(t^2) dt. F'(4) = e^((4^3)^2) * 3(4)^2 = 48e^4096. Too large. Let's use F(x) = int(0 to x^2) sin(t) dt. F'(x) = sin(x^2)*2x. F'(pi) = sin(pi^2)*2pi. Let's use F(x) = int(1 to sqrt(x)) f(t) dt. F'(4) = f(2) * 1/4. Let's just use F(x) = int(0 to x^2) e^t dt. F'(1) = e^1 * 2 = 2e. I will update stem in script.

82. Answer: A — Avg val = 3. 3 = c^2 => c = sqrt(3).

83. Answer: C — Let's fix the stem to $\int_0^{\ln 2} 2e^x \, dx = 2$.

84. Answer: B — Let u = ln x. Integral of u du = (ln x)^2 / 2. Eval 1 to e = 1/2.

85. Answer: A — sin(x^2)*2x - sin(x)*1.

86. Answer: C — int(0 to 1) 2x dx + int(1 to 2) (x^2+1) dx = 1 + (7/3 + 1) = 11/3.

87. Answer: A — Total area = 2sqrt(2) - 2.

88. Answer: A — g' = f. g'' = f'. g concave up when f' > 0.

89. Answer: B — 100 + integral(0 to 3) (10 - t^2) dt = 109.

90. Answer: A — Left sum: f(0) + f(1) + f(2) + f(3) = 0 + 1 + 4 + 9 = 14.

Mini Practice Test 1 — MCQ

1. *lim*(x\u21923) (x^{2} \u2212 9) / (x \u2212 3) = *[No Calculator]*

(A) 0

(B) 3

(C) 6

(D) Does not exist

2. *lim*(x\u21920) sin(4x) / x = *[No Calculator]*

(A) 0

(B) 1

(C) 4

(D) Does not exist

3. *lim*(x\u21920) (1 \u2212 cos(x)) / x^{2} = *[No Calculator]*

(A) 0

(B) 1/2

(C) 1

(D) Does not exist

4. The horizontal asymptote of y = (2x \u2212 1) / (x + 3) is *[No Calculator]*

(A) y = 2

(B) y = 1

(C) y = 3

(D) y = –1

5. Let f(x) = x^{3} \u2212 x \u2212 1. The IVT guarantees a root on which interval? *[No Calculator]*

(A) [0, 1]

(B) [1, 2]

(C) [2, 3]

(D) [–1, 0]

6. If f(x) = { x^{2} + k for x \u2264 1; 2x for x > 1 } is continuous at x=1, then k = *[No Calculator]*

(A) 0

(B) 1

(C) 2

(D) 3

7. *lim*(x\u21920) x^{2} sin(1/x) = *[No Calculator]*

(A) 0

(B) 1

(C) Does not exist

(D) ∞

8. The expression *lim*(h\u21920) [f(2+h) \u2212 f(2)] / h represents *[No Calculator]*

(A) f(2)

(B) f(x)

(C) f′(x)

(D) f′(2)

9. If f(x) = $3x^4$ \u2212 5x, then f\u2032(x) = *[No Calculator]*

(A) $12x^3$

(B) $12x^3 – 5$

(C) $3x^3 – 5$

(D) $12x^3 + 5$

10. If y = x^2 sin(x), then dy/dx = *[No Calculator]*

(A) 2x cos(x)

(B) x^2 cos(x)

(C) 2x sin(x) + x^2 cos(x)

(D) 2x sin(x) – x^2 cos(x)

11. If f(x) = x / (x^2 + 1), then f\u2032(x) = *[No Calculator]*

(A) (1 – x^2) / (x^2 + 1)^2

(B) ['(1 + x^2) / (x^2 + 1)^2']

(C) (x^2 – 1) / (x^2 + 1)^2

(D) $2x^2$ / (x^2 + 1)^2

12. If f(x) = sin(x^3), then f\u2032(x) = *[No Calculator]*

(A) cos(x^3)

(B) $3x^2$ cos(x^3)

(C) $3x^2$ sin(x^3)

(D) x^3 cos(x^3)

13. For the curve x^2 + y^2 = 25, dy/dx = *[No Calculator]*

(A) –x/y

(B) x/y

(C) –y/x

(D) y/x

14. d/dx [ln(3x)] = *[No Calculator]*

(A) 1/x

(B) 3/x

(C) 1/(3x)

(D) x

15. If f(x) = e^{2x}, then f\u2032(x) = *[No Calculator]*

(A) e^{2x}

MINI PRACTICE TEST 3

Full Cumulative

20 Questions | 45 Minutes | Mixed Calculator

Mini Practice Test 3 — MCQ

1. *lim*(x\u21922) (x^{3} \u2212 8) / (x^{2} \u2212 4) = *[Mixed]*

(A) 0

(B) Does not exist

(C) 3

(D) 4

2. The function f(x) = 1/(x \u2212 2) has what type of discontinuity at x = 2? *[Mixed]*

(A) Removable

(B) Infinite

(C) Jump

(D) None

3. *lim*(x\u21920) (\u221a(x + 4) \u2212 2) / x = *[Mixed]*

(A) 1/4

(B) 1/2

(C) 0

(D) 1

4. If f(x) = x^{3} \u2212 4x, then f\u2032(x) = *[Mixed]*

(A) 3x^{2}

(B) 3x^{2} – 4

(C) x^{4}/4 – 2x^{2}

(D) 3x^{2} + 4

5. d/dx [e^{x} cos(x)] = *[Mixed]*

(A) e^{x} sin(x)

(B) –e^{x} sin(x)

(C) e^{x}(cos(x) – sin(x))

(D) e^{x}(cos(x) + sin(x))

6. For x y = 6, dy/dx = *[Mixed]*

(A) y/x

(B) –y/x

(C) –x/y

(D) 6/x^{2}

7. d/dx [ln(cos(x))] = *[Mixed]*

(A) tan(x)

(B) sec(x)

(C) 1/cos(x)

(D) –tan(x)

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