ALGEBRA 2

Polynomial & Rational Functions

Complete Unit: Visual Guided Notes

Grades 10 – 11 | 6-Week Unit | 48 Pages

Topics Covered:

- Polynomial Operations (Add, Subtract, Multiply, Divide)
- Factoring Strategies (GCF, Grouping, Sum/Difference of Cubes)
- End Behavior & Graphing Polynomial Functions
- Remainder Theorem & Factor Theorem
- Rational Expressions (Simplify, Multiply, Divide, Add, Subtract)
- Graphing Rational Functions (Asymptotes & Holes)
- Real-World Polynomial Modeling Applications

Includes: Guided Notes • 50 Practice Problems • Graphing Activities
40 Rational Drills • Discovery Activities • Mini-Project
2 Quizzes • Unit Test • Fully Worked Answer Keys

UM Printables

Table of Contents

§1 Polynomial Operations

What Is a Polynomial?

Definition: Polynomial

A polynomial is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where n is a non-negative integer and each a_i is a real number called a coefficient. The exponents must be non-negative integers.

Degree: The highest exponent in the polynomial (when written in standard form).

Leading Coefficient: The coefficient of the term with the highest degree.

Constant Term: The term without a variable (a_0).

Standard Form: Terms arranged in descending order of degree.

Classification by Number of Terms: Monomial (1 term) | Binomial (2 terms) | Trinomial (3 terms)

Classification by Degree: Constant (0) | Linear (1) | Quadratic (2) | Cubic (3) | Quartic (4) | Quintic (5)

Examples — Identify the degree, leading coefficient, and constant term.

$3x^4 - 2x^3 + 7x - 5$ → Degree: 4, Leading Coeff: 3, Constant: –5

$x^2 + 5$ → Degree: 2, Leading Coeff: 1, Constant: 5

-7 → Degree: 0 (constant polynomial), Leading Coeff: –7, Constant: –7

Non-Examples — These are NOT polynomials.

$x^{-2} + 3$ → Negative exponent

$\sqrt{x} + 1$ → Fractional exponent ($x^{1/2}$)

$2/x + 3$ → Variable in denominator (equals $2x^{-1} + 3$)

YOU TRY!

For $5x^3 - 2x^7 + 4$, find the degree, leading coefficient, and constant term.

Is $\sqrt{3} \cdot x^2 + x - 1$ a polynomial? Explain why or why not.

Classify $4x^3 - 1$ by degree and by number of terms.

Adding & Subtracting Polynomials

Key Principle

To add or subtract polynomials, combine like terms (terms with the same variable raised to the same power). When subtracting, distribute the negative sign to every term inside the parentheses.

Example 1: Add

Step 1: $(3x^2 + 2x - 1) + (5x^2 - 3x + 4)$

Step 2: Remove parentheses: $3x^2 + 2x - 1 + 5x^2 - 3x + 4$

Step 3: Group like terms: $(3x^2 + 5x^2) + (2x - 3x) + (-1 + 4)$

Step 4: Combine: $\mathbf{8x^2 - x + 3}$

Example 2: Subtract

Step 1: $(2x^3 - x + 7) - (x^3 + 4x - 3)$

Step 2: Distribute the negative: $2x^3 - x + 7 - x^3 - 4x + 3$

Step 3: Group like terms: $(2x^3 - x^3) + (-x - 4x) + (7 + 3)$

Step 4: Combine: $\mathbf{x^3 - 5x + 10}$

Common Error: When subtracting polynomials, the negative sign must be distributed to **every** term. A common mistake is: $-(x^2 + 3x - 2) = -x^2 + 3x - 2$ **WRONG** | Correct: $-x^2 - 3x + 2$ **CORRECT**

Example 3: Subtract (vertical format)

Step 1: Align like terms vertically for subtraction:

Step 2: $4x^3 - 2x^2 + 5x - 8$

Step 3: $-(x^3 + 3x^2 - x + 2)$

Step 4: Distribute negative, then combine:

Step 5: $4x^3 - 2x^2 + 5x - 8$

Step 6: $- x^3 - 3x^2 + x - 2$

Step 7: $\mathbf{3x^3 - 5x^2 + 4x - 10}$

YOU TRY!

$(4x^3 - 2x + 5) + (x^3 + 7x - 1)$

$(6x^2 - 3x + 8) - (2x^2 + 5x - 4)$

$(x^4 + 2x^2 - 3) + (3x^4 - x^2 + 7)$

Multiplying Polynomials

Three key methods, depending on the number of terms:

Method 1: Monomial × Polynomial (Distributive Property)

Distribute the monomial to each term of the polynomial.

Formula: $a(b + c) = ab + ac$

Example 4: Monomial × Polynomial

Step 1: $3x^2(2x^3 - 4x + 1)$

Step 2: Distribute: $3x^2 \cdot 2x^3 + 3x^2 \cdot (-4x) + 3x^2 \cdot 1$

Step 3: Multiply coefficients and add exponents: $\mathbf{6x^5 - 12x^3 + 3x^2}$

Method 2: Binomial × Binomial (FOIL)

First · **O**uter · **I**nner · **L**ast

Then combine like terms.

Example 5: FOIL

Step 1: $(x + 3)(x - 5)$

Step 2: F: $x \cdot x = x^2$

Step 3: O: $x \cdot (-5) = -5x$

Step 4: I: $3 \cdot x = 3x$

Step 5: L: $3 \cdot (-5) = -15$

Step 6: Combine: $x^2 - 5x + 3x - 15 = \mathbf{x^2 - 2x - 15}$

Method 3: Special Products — Memorize These!

Square of a Sum: $(a + b)^2 = a^2 + 2ab + b^2$

Square of a Difference: $(a - b)^2 = a^2 - 2ab + b^2$

Difference of Squares: $(a + b)(a - b) = a^2 - b^2$

Example 6: Special Products

Step 1: $(2x + 5)^2 = (2x)^2 + 2(2x)(5) + 5^2 = \mathbf{4x^2 + 20x + 25}$

Step 2: $(3x - 4)^2 = (3x)^2 - 2(3x)(4) + 4^2 = \mathbf{9x^2 - 24x + 16}$

Step 3: $(x + 7)(x - 7) = x^2 - 7^2 = \mathbf{x^2 - 49}$

Method 4: Larger Polynomials — Distribute Each Term

Multiply each term of the first polynomial by each term of the second, then combine like terms.

Example 7: Binomial × Trinomial

Step 1: $(x + 2)(x^2 - 3x + 4)$

Step 2: Distribute x: $x^3 - 3x^2 + 4x$

Step 3: Distribute 2: $+ 2x^2 - 6x + 8$

Step 4: Combine: $x^3 + (-3x^2 + 2x^2) + (4x - 6x) + 8$

Step 5: $\mathbf{x^3 - x^2 - 2x + 8}$

YOU TRY!

$4x(3x^2 - 2x + 7)$

$(x + 6)(x - 4)$

$(2x - 3)^2$

$(x + 1)(x^2 - 5x + 3)$

Dividing Polynomials

Method 1: Long Division

Works for dividing **any** polynomial by another polynomial. Follow the same process as numeric long division: divide, multiply, subtract, bring down, repeat.

Division Algorithm for Polynomials

Dividend = Divisor × Quotient + Remainder
If $P(x)$ is divided by $D(x)$: $P(x) = D(x) \cdot Q(x) + R(x)$
where the degree of $R(x)$ is less than the degree of $D(x)$, or $R(x) = 0$.

Example 8: Long Division

Step 1: Divide $(2x^3 + 3x^2 - 5x + 7)$ by $(x + 2)$

Step 2: Divide $2x^3 \div x = 2x^2$. Multiply: $2x^2(x + 2) = 2x^3 + 4x^2$. Subtract: $(3x^2 - 4x^2) = -x^2$. Bring down $-5x$.

Step 3: Divide $-x^2 \div x = -x$. Multiply: $-x(x + 2) = -x^2 - 2x$. Subtract: $(-5x + 2x) = -3x$. Bring down $+7$.

Step 4: Divide $-3x \div x = -3$. Multiply: $-3(x + 2) = -3x - 6$. Subtract: $7 - (-6) = 13$.

Step 5: **Quotient: $2x^2 - x - 3$, Remainder: 13**

Method 2: Synthetic Division

A shortcut for dividing a polynomial by a **linear binomial** of the form $(x - c)$. It is faster and less error-prone than long division for this special case.

Synthetic Division Steps

1. Write c (the zero of the divisor) on the left.
2. Write the coefficients of the dividend in order (include 0 for missing terms).
3. Bring down the first coefficient.
4. Multiply by c and write the result under the next coefficient.
5. Add the column and write the sum below the line.
6. Repeat steps 4–5 until all columns are complete.
7. The last number is the remainder. The others are the quotient coefficients (degree is one less than the dividend).

Example 9: Synthetic Division

Step 1: Divide $(2x^3 + 3x^2 – 5x + 7)$ by $(x + 2)$

Step 2: Divisor: $x + 2 = x – (–2)$, so $c = –2$

Step 3: Coefficients: 2 3 –5 7

Step 4: Bring down 2. Multiply $–2 \times 2 = –4$. Add $3 + (–4) = –1$.

Step 5: Multiply $–2 \times (–1) = 2$. Add $–5 + 2 = –3$.

Step 6: Multiply $–2 \times (–3) = 6$. Add $7 + 6 = 13$.

Step 7: **Quotient: $2x^2 – x – 3$, Remainder: 13** (same as long division!)

Tip: If the divisor is $(ax – b)$ with $a \neq 1$, you can still use synthetic division with $c = b/a$, but you must divide your final quotient by a. Alternatively, use long division.

Common Error: Always include 0 coefficients for missing terms in synthetic division! For $2x^4 – x + 3$, the coefficient row is: 2, 0, 0, –1, 3 (the x^3 and x^2 terms are missing).

YOU TRY!

Divide $(x^3 + 4x^2 – 7x + 6)$ by $(x + 1)$ using long division.

Divide $(3x^3 – 2x^2 + 4x – 1)$ by $(x – 2)$ using synthetic division.

Divide $(x^4 + 0x^3 – 3x + 5)$ by $(x + 3)$ using synthetic division. (Remember the zero placeholders!)

---

§2 Factoring Strategies

Greatest Common Factor (GCF)

GCF Factoring

The first step in any factoring problem is to check for a GCF. Factor out the largest factor that divides into every term of the polynomial (both the numerical GCF and the variable GCF with the smallest exponent).

Example 10: Factor out the GCF

Step 1: $12x^3 + 8x^2 - 4x$

Step 2: Numerical GCF of 12, 8, 4 is 4. Variable GCF: x (smallest exponent is 1).

Step 3: GCF = $4x$

Step 4: $12x^3 \div 4x = 3x^2$, $8x^2 \div 4x = 2x$, $-4x \div 4x = -1$

Step 5: $\mathbf{4x(3x^2 + 2x - 1)}$

Example 11: GCF with Grouping Inside

Step 1: $5x(x + 2) + 3(x + 2)$

Step 2: The common binomial factor is $(x + 2)$.

Step 3: $\mathbf{(x + 2)(5x + 3)}$

Tip: When the leading coefficient is negative, factor out a negative GCF so that the remaining polynomial has a positive leading coefficient. This makes subsequent factoring easier.

YOU TRY!

$18x^4 - 12x^2$

$-6x^3 + 9x^2 - 3x$

$7y(y - 4) - 2(y - 4)$

Horizontal & Oblique Asymptotes

While vertical asymptotes describe side-to-side behavior, horizontal and oblique asymptotes describe the end behavior (what happens as $x \to \pm\infty$).

Rules for Horizontal Asymptotes (H.A.)

Let n = degree of numerator, d = degree of denominator.

If n < d: H.A. at y = 0 (x-axis).
If n = d: H.A. at y = (leading coeff of num) / (leading coeff of den).
If n > d: There is NO horizontal asymptote. (Check for oblique asymptote!)

Example 42: Horizontal Asymptotes

Step 1: $f(x) = (2x^2 + 1) / (5x^2 - 3) \to n=2, d=2$ (n=d). H.A. at $y = 2/5$.

Step 2: $g(x) = (3x + 2) / (x^2 - 4) \to n=1, d=2$ (n<d). H.A. at $y = 0$.

Step 3: $h(x) = (x^3 - 1) / (2x - 7) \to n=3, d=1$ (n>d). No H.A.

Oblique (Slant) Asymptotes

An oblique asymptote exists if the degree of the numerator is **exactly one more** than the degree of the denominator ($n = d + 1$).

How to find: Divide the numerator by the denominator using long division or synthetic division. The quotient (ignoring the remainder) is the equation of the asymptote, $y = Q(x)$.

Example 43: Oblique Asymptote

Step 1: $f(x) = (x^2 + 4x + 5) / (x + 2)$

Step 2: Degrees: n=2, d=1. $n = d + 1$, so there is an oblique asymptote.

Step 3: Long Division: $(x^2 + 4x + 5) \div (x + 2) = x + 2$, R 1.

Step 4: The oblique asymptote is $\mathbf{y = x + 2}$.

YOU TRY!

Find the horizontal asymptote of $f(x) = (4x^3 + 2x) / (5x^3 – 1)$.

Find the horizontal asymptote of $f(x) = (7x + 3) / (2x^2 + 9)$.

Find the oblique asymptote of $f(x) = (2x^2 – 3x + 1) / (x – 4)$.

Level 2: Intermediate (Problems 18–34)

1. 18. $(3x^3 - 2x^2 + 4) + (-x^3 + 5x^2 - x - 8)$

2. 19. $(5x^4 - 3x^2 + 1) - (2x^4 - x^3 + 3x^2 - 4)$

3. 20. $-2x^2(3x^2 - 5x + 1)$

4. 21. $(2x - 3)(4x^2 + x - 5)$

5. 22. $(x^2 + 3)(x^2 - 3)$

6. 23. $(3x + 1)^3$ (Hint: square first, then multiply)

7. 24. Divide $(3x^3 + 5x^2 - 11x + 3)$ by $(x + 3)$

8. 25. Divide $(2x^3 - 3x^2 + 4x - 1)$ by $(x - 2)$

9. 26. Divide $(4x^3 + 5x^2 - 2x + 7)$ by $(x + 1)$

10. 27. Divide $(x^4 - 16)$ by $(x + 2)$

11. 28. Divide $(x^3 + 8)$ by $(x + 2)$

12. 29. $(x^3 - 2x^2 + 5x) - (3x^3 - 4x + 2) + (x^2 - x)$

13. 30. $(2x + 5)(2x - 5)(x^2 + 3)$

14. 31. Divide $(6x^3 - 7x^2 + 5)$ by $(2x - 1)$ (Hint: use long division)

15. 32. Divide $(9x^3 - 1)$ by $(3x - 1)$

16. 33. $(3x^2 - x + 4)(2x^2 + 5)$

17. 34. Divide $(2x^4 + 3x^3 – 5x + 2)$ by $(x^2 + 1)$ (Long division)

Level 3: Advanced (Problems 35–50)

1. 35. $[(x + 3)^2 – (x – 2)^2] \div 5$ (Simplify first, then divide)

2. 36. $(x^3 – 27) \div (x – 3)$

3. 37. Divide $(x^4 + 0x^3 – 5x^2 + 0x + 4)$ by $(x – 2)$ (Use synthetic)

4. 38. $(2x – 1)^4$

5. 39. Divide $(3x^4 + 2x^3 – 8x + 6)$ by $(x + 2)$ (Synthetic)

6. 40. $[(x^2 – 4)(x + 3)] \div [(x – 2)(x^2 + 3x – 4)]$ (Factor, then simplify)

7. 41. $(x^5 – 1) \div (x – 1)$ (Synthetic division)

8. 42. Divide $(2x^3 + 9x^2 + 10x – 8)$ by $(2x + 3)$ (Long division or synthetic with adjustment)

9. 43. Evaluate $P(–2)$ using synthetic division for $P(x) = x^4 – 3x^2 + 5x – 1$

10. 44. $(x^2 + 4)^2 – (x^2 – 4)^2$

11. 45. Divide $(6x^4 – x^3 + 2x – 7)$ by $(2x^2 – x + 1)$ (Long division)

12. 46. If $P(x) = 3x^3 – 2x^2 + 4$ is divided by $(x – k)$, the remainder is 40. Find k.

13. 47. Divide $(x^6 – 64)$ by $(x^2 – 4)$ (Hint: treat x^2 as a single variable or factor difference of squares twice)

14. 48. $(3x^2 – 5x + 2)^2$

15. 49. Divide $(x^4 – 2x^3 + x – 5)$ by $(x^2 – x + 2)$

Thank You!

Your support means the world to me!

How was this resource?

I pour my heart into creating high-quality, rigorous, and visually engaging materials for your classroom. Your feedback helps me improve and allows other teachers to find these resources!

★ ★ ★ ★ ★

Please take a moment to leave a review on Teachers Pay Teachers!

UM Printables

Empowering Teachers, Engaging Students.