

VOLUME, SURFACE AREA & 3D GEOMETRY

Complete Unit

Grades 7 - 10

- Prisms (Triangular, Rectangular, Trapezoidal, Hexagonal)
- Cylinders & Hollow Cylinders
- Pyramids & Frustum of a Pyramid
- Cones & Frustum of a Cone
- Spheres & Hemispheres
- Composite Figures & Real-World Applications
- Cavalieri's Principle & Cross-Section Analysis
- Density Applications ($\text{Mass} = \text{Density} \times \text{Volume}$)

What's Inside:

Visual Guided Notes with 3D Diagrams & Nets

3D Net Identification & Folding Activity (10 Nets)

40 Surface Area Practice Problems with Full Solutions

40 Volume Practice Problems with Full Solutions

Cavalieri's Principle & Cross-Section Discovery Activity

25 Real-World Application Problems

10 Density & Mass Application Problems

Cumulative Unit Review

UM Printables

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- Volume: $V = \pi \times r^2 \times h$
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- Nets of Pyramids
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Section 4: Cones

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Activity: Cavalieri's Principle & Cross-Sections

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- 10 Nets with Labelled Faces and Dimensions

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Practice: Volume Practice Set (40 Problems)

Applications: Real-World Application Problems (25)

- Packaging, Silos, Architecture, Material Cost

Applications: Density & Mass Problems (10)

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SECTION 1

Prisms

Triangular, Rectangular, Trapezoidal & Hexagonal

1.1 What is a Prism?

A **prism** is a three-dimensional solid with two identical, parallel bases connected by rectangular lateral faces. The shape of the base gives the prism its name. All cross-sections parallel to the bases are congruent to the base.

Key Vocabulary:

Term	Definition
Base (B)	The two identical, parallel faces of the prism
Height (h)	Perpendicular distance between the two bases
Lateral Face	A rectangular face connecting the two bases
Lateral Edge	The edge where two lateral faces meet
Right Prism	Lateral edges are perpendicular to the bases
Oblique Prism	Lateral edges are NOT perpendicular to the bases

Note: In this unit, we focus on **right prisms** where all lateral faces are rectangles. The prism height equals the length of any lateral edge.

1.2 Types of Prisms

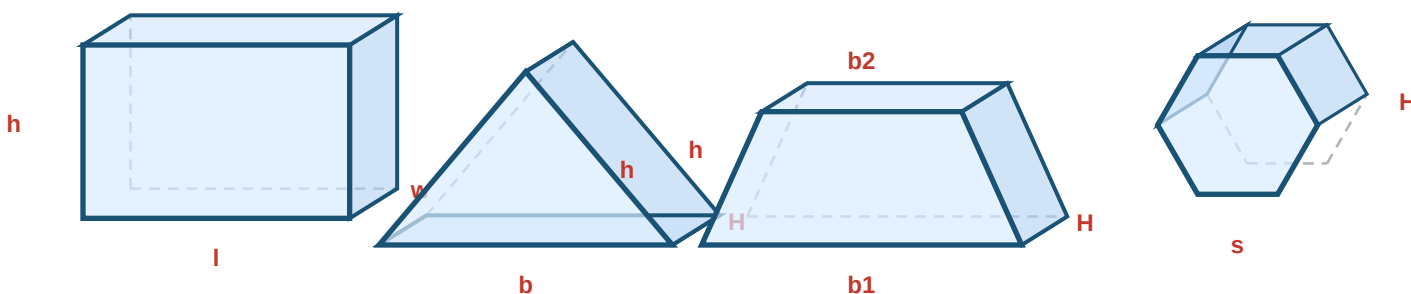
A prism is named by the shape of its base:

Rectangular

Triangular

Trapezoidal

Hexagonal



1.3 Lateral Surface Area of a Prism

The **lateral surface area (LSA)** is the total area of all the rectangular side faces. Picture "unrolling" the sides into a single rectangle: its width equals the perimeter of the base and its height equals the prism height.

$$\text{LSA} = P \times h \text{ (} P = \text{base perimeter, } h = \text{prism height)}$$

1.4 Total Surface Area of a Prism

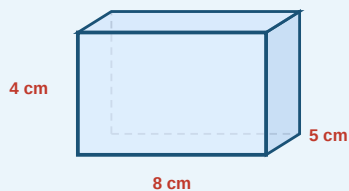
The **total surface area (TSA)** includes the two bases plus all lateral faces:

$$\text{TSA} = \text{LSA} + 2B \text{ (} B = \text{area of one base)}$$

Prism	Base Area B	Perimeter P	TSA = Ph + 2B
Rectangular	$l \times w$	$2(l+w)$	$2lw + 2lh + 2wh$
Triangular	$(1/2) b \times h_3$	$a + b + c$	$(a+b+c)H + bh_3$
Trapezoidal	$(1/2)(b_1+b_2)h_3$	$a+b_1+c+b_2$	$(P)H + (b_1+b_2)h_3$
Hexagonal (reg.)	$(3\sqrt{3}/2)s^2$	$6s$	$6sH + 3\sqrt{3}s^2$

Worked Example 1: Rectangular Prism Surface Area

A rectangular prism has length = 8 cm, width = 5 cm, height = 4 cm. Find the LSA and TSA.



$l=8, w=5, h=4$ cm

Solution:

Step 1: Identify: $l=8, w=5, h=4$

Step 2: Perimeter: $P = 2(8+5) = 26$ cm

Step 3: $\text{LSA} = P \times h = 26 \times 4 = 104 \text{ cm}^2$

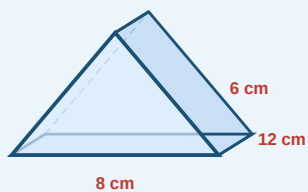
Step 4: Base area: $B = 8 \times 5 = 40 \text{ cm}^2$

Step 5: $\text{TSA} = \text{LSA} + 2B = 104 + 80 = 184 \text{ cm}^2$

Check: $2(8 \times 5) + 2(8 \times 4) + 2(5 \times 4) = 80 + 64 + 40 = 184 \text{ cm}^2$ (check)

Worked Example 2: Triangular Prism Surface Area

A right-triangular prism has legs 6 cm and 8 cm (hypotenuse 10 cm) and prism height $H = 12$ cm. Find the TSA.



legs 6, 8 cm; hyp 10 cm; H=12 cm

Solution:

Step 1: Base area: $B = (1/2) \times 6 \times 8 = 24 \text{ cm}^2$

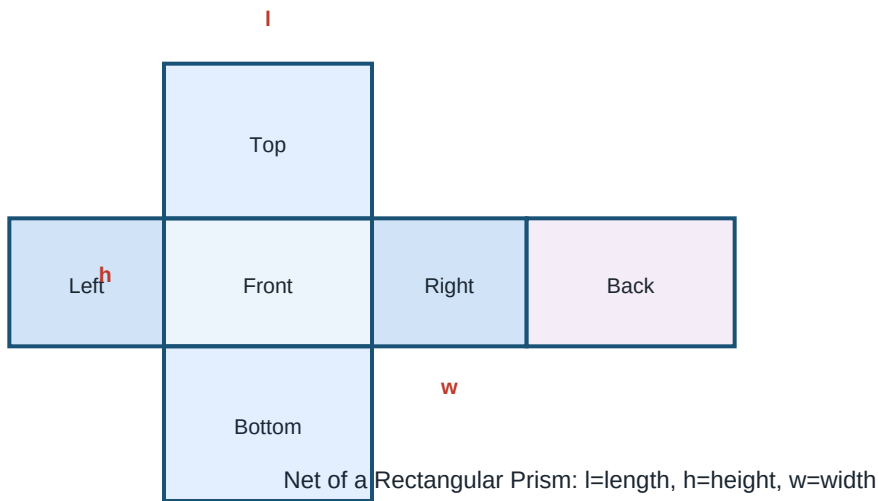
Step 2: Perimeter: $P = 6 + 8 + 10 = 24 \text{ cm}$

Step 3: LSA = $P \times H = 24 \times 12 = 288 \text{ cm}^2$

Step 4: TSA = $288 + 2(24) = 336 \text{ cm}^2$

1.5 Nets of Prisms

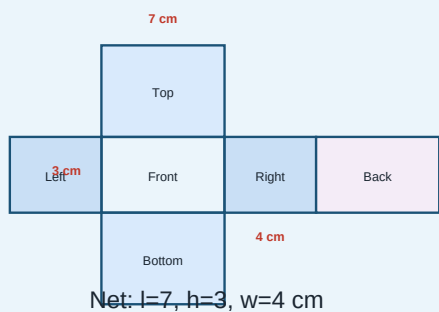
A **net** is a flat pattern that folds up to form a 3D solid. The total surface area equals the sum of all face areas in the net. Study the net for the rectangular prism below -- notice how each face unfolds.



Note: When you fold a net, edges that touch become the edges of the 3D solid. Each face in the net corresponds to exactly one face of the prism.

Worked Example 3: Using a Net to Find TSA

A rectangular prism has dimensions 7 cm x 4 cm x 3 cm. Use the net to find the TSA.



Solution:

Step 1: Front & Back (7×3): $2 \times 21 = 42 \text{ cm}^2$

Step 2: Top & Bottom (7×4): $2 \times 28 = 56 \text{ cm}^2$

Step 3: Left & Right (4×3): $2 \times 12 = 24 \text{ cm}^2$

Step 4: TSA = $42 + 56 + 24 = 122 \text{ cm}^2$

1.6 Volume of a Prism

SECTION 2

Cylinders

Lateral & Total Surface Area, Volume & Hollow Cylinders

2.1 What is a Cylinder?

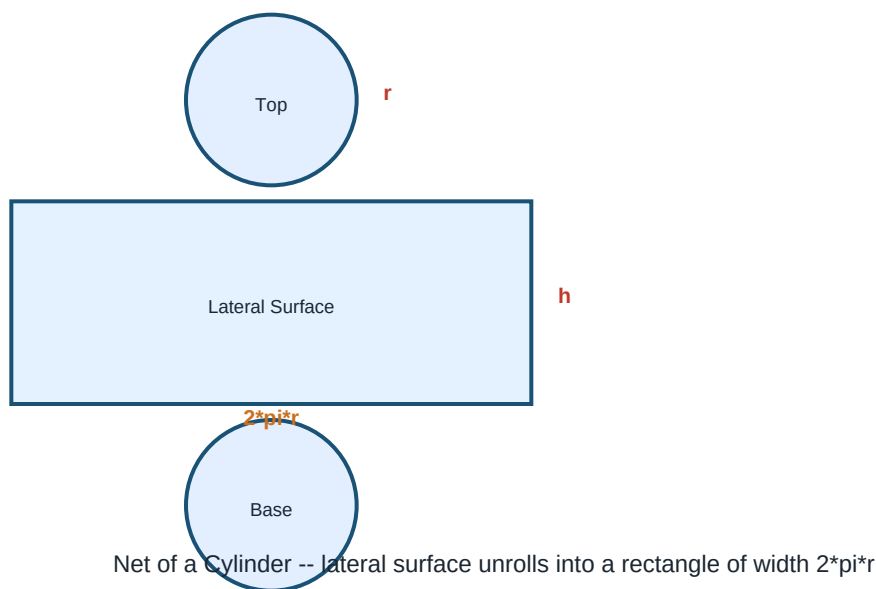
A **cylinder** is a solid with two identical circular bases connected by a curved lateral surface. In a **right circular cylinder**, the axis joining the two base centers is perpendicular to the bases.

Key Vocabulary:

Term	Definition
Radius (r)	Distance from the center to the edge of the circular base
Diameter (d)	Distance across the base through the center; $d = 2r$
Height (h)	Perpendicular distance between the two circular bases
Axis	Imaginary line joining the centers of both bases

2.2 Surface Area of a Cylinder

When you "unroll" the curved surface, it forms a rectangle with width = circumference = $2\pi r$ and height = h .



$$LSA = 2 * \pi * r * h$$

$$TSA = 2\pi r^2 + 2\pi r h = 2\pi r(r + h)$$

Note: Each circular base has area πr^2 . Two bases = $2\pi r^2$. The rectangle (lateral surface) has area $2\pi r \times h$.

Worked Example 1: Cylinder Surface Area

A cylinder has $r=5$ cm, $h=12$ cm. Find LSA and TSA. Give answers in terms of π and rounded.

SECTION 3

Pyramids

Surface Area, Volume, Nets & Frustum of a Pyramid

3.1 What is a Pyramid?

A **pyramid** has one polygonal base and triangular lateral faces meeting at a point called the **apex**. Pyramids are named by their base shape (square pyramid, triangular pyramid, etc.).

Key Vocabulary:

Term	Definition
Apex	The single vertex where all lateral faces meet
Height (h)	Perpendicular distance from the apex down to the base
Slant Height (l)	Height of one lateral triangular face (apex to midpoint of a base edge)

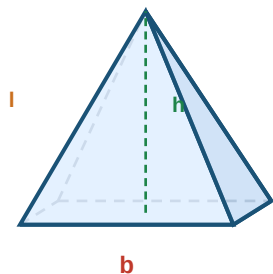
Note: Key distinction: The vertical height h goes straight down through the center. The slant height l runs along the surface of a triangular face. They are NOT interchangeable -- use h for volume and l for lateral surface area.

3.2 Surface Area of a Pyramid

For a **regular pyramid** all lateral triangular faces are identical. Each has area = $(1/2) \times \text{base-edge} \times \text{slant-height}$.

$$\text{LSA} = (1/2) \times P \times l \quad (P = \text{base perimeter}, l = \text{slant height})$$

$$\text{TSA} = (1/2) \times P \times l + B \quad (B = \text{base area})$$



Square Pyramid: vertical height h (green dashed), slant height l (orange)

Worked Example 4: Square Pyramid Surface Area

SECTION 6

Composite Figures

Identifying, Decomposing & Real-World Problems

6.1 Decomposing Composite Solids

Most real-world objects are not simple solids -- they are composites. The strategy:

Step 1: Identify the simple solids that make up the composite.

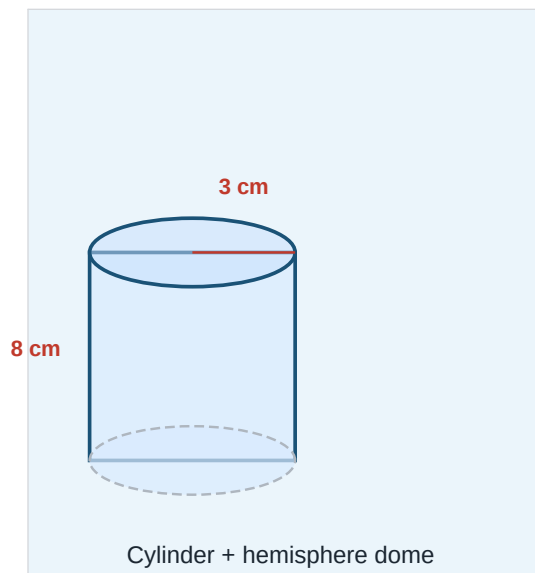
Step 2: Find the volume or surface area of each part.

Step 3: Combine -- add volumes; for SA, add all areas then subtract any hidden (internal) joining faces.

***Note:** When two solids are joined, the touching faces are no longer on the surface. Subtract twice that area (once from each solid) from the total surface area.*

Worked Example 1: Cylinder + Hemisphere (Ice Cream Cone Scoop)

A cylindrical container ($r=3$ cm, $h=8$ cm) is topped with a hemisphere ($r=3$ cm). Find total volume and outer surface area (in terms of π).

 Cylinder + hemisphere dome	Solution: Volume: $V_{\text{cyl}} = \pi r^2 h = \pi 3^2 8 = 72\pi \text{ cm}^3$ $V_{\text{hemi}} = \frac{2}{3} \pi r^3 = \frac{2}{3} \pi 27 = 18\pi \text{ cm}^3$ $V_{\text{total}} = 72\pi + 18\pi = 90\pi \text{ cm}^3$ Surface Area: $\text{Cyl LSA} = 2\pi r h = 2\pi 3 8 = 48\pi \text{ (no top -- joined with hemisphere)}$ $\text{Cyl base} = \pi r^2 = \pi 9 = 9\pi$ $\text{Hemi curved SA} = 2\pi r^2 = 2\pi 9 = 18\pi$ $\text{TSA} = 48\pi + 9\pi + 18\pi = 75\pi \text{ cm}^2$
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Worked Example 2: Hollowed Solid

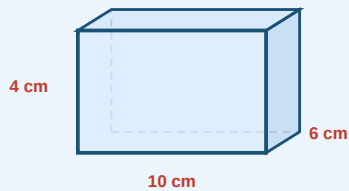
A sphere of $r=6$ cm has a cylindrical hole ($r=3$, $h=6$ cm) drilled through its center. Find the remaining volume (in terms of π).

Volume Practice Set

Find the volume for each solid. Show formula, substitution, and final answer. Diagrams not drawn to scale.

Problem 1: Rectangular Prism

V: $l=10$, $w=6$, $h=4$ cm.

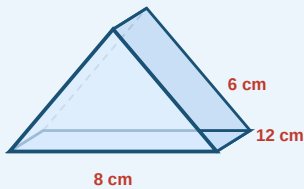


Solution:

$$V = lwh = 10 \times 6 \times 4 = \mathbf{240 \text{ cm}^3}$$

Problem 2: Triangular Prism

V: right tri base (legs 6, 8, hyp 10). $H=12$ cm.



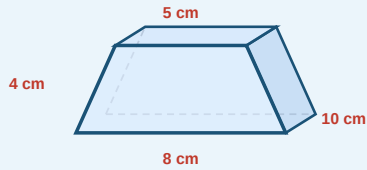
Solution:

$$B = \frac{1}{2}(6)(8) = 24 \text{ cm}^2$$

$$V = 24 \times 12 = \mathbf{288 \text{ cm}^3}$$

Problem 3: Trapezoidal Prism

V: $b_1=8$, $b_2=5$, $h=4$ cm. Prism $H=10$ cm.



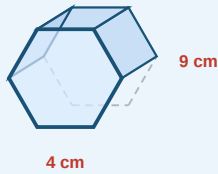
Solution:

$$B = \frac{1}{2}(8+5)(4) = 26 \text{ cm}^2$$

$$V = 26 \times 10 = \mathbf{260 \text{ cm}^3}$$

Problem 4: Hexagonal Prism

V: $s=4$ cm, $H=9$ cm. (Leave in terms of $\sqrt{3}$)



Solution:

$$B = \frac{3\sqrt{3}}{2}(16) = 24\sqrt{3} \text{ cm}^2$$

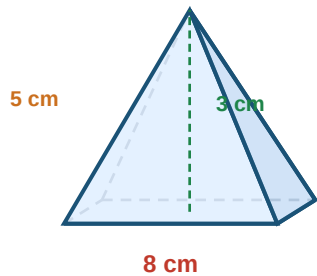
$$V = 24\sqrt{3} \times 9 = \mathbf{216\sqrt{3} \text{ cm}^3}$$

Unit Quiz 2: Pyramids, Cones & Spheres

Directions: Calculate volume and surface area. Leave answers in terms of pi unless told to round. Show all work.

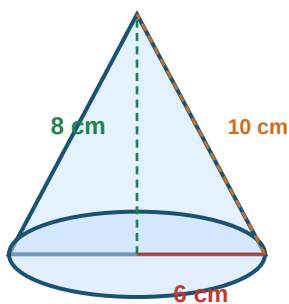
1. Square Pyramid

Volume and TSA. Base edge=8 cm, height=3 cm.



2. Cone

Volume and TSA. $r=6$ cm, $h=8$ cm.



FULL RESOURCE AT A GLANCE

VOLUME, SURFACE AREA & 3D GEOMETRY • COMPLETE UNIT

107-PAGE UNIT

Grades 7–10 | Guided instruction, practice, applications, review, and assessment

CONTENT & GEOMETRY

- 6 sections: Prisms, Cylinders, Pyramids, Cones, Spheres, and Composite Figures
- Lateral and total surface area + volume
- Hollow cylinders, pyramid/cone frustums, hemispheres
- Composite solids, decomposition, and real-world geometry

VISUAL ACTIVITIES

- 3D diagrams, nets, and guided notes
- 3D Net Identification & Folding Activity - 10 nets
- Cavalieri's Principle & Cross-Section Discovery Activity
- Cross-section identification and volume reasoning

PRACTICE & APPLICATIONS

- 40 Surface Area Practice Problems with solutions
- 40 Volume Practice Problems with solutions
- 10 Real-World Application Problems
- 10 Density & Mass Application Problems

REVIEW & ASSESSMENT

- Cumulative Unit Review
- Unit Quiz 1 - Prisms & Cylinders
- Unit Quiz 2 - Pyramids, Cones & Spheres
- Comprehensive Unit Test
- Solutions included throughout the resource

UM Printables

UMPrintables.com

A complete 107-page unit resource covering instruction, practice, applications, review, and assessment.