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GRADE 8 MATHEMATICS

Slope & Proportional Relationships

A Complete Unit

Connecting Lines & Linear Equations

1

Proportional Relationships
& Unit Rate as Slope

2

Slope of a
Non-Vertical Line

3

Similar Triangles
& Slope

4

Equations
of Lines

5

Comparing
Proportional
Relationships

Aligned to CCSS-M Standards

8.EE.B.5 · 8.EE.B.6 · 8.F.A.2 · 8.F.A.3

Proportional Relationships & Unit Rate as Slope

The line $y = mx$ through the origin — and what its slope really means

Learning Targets

Students will be able to:

- Graph a proportional relationship $y = mx$ and explain why it passes through the origin.
- Identify the unit rate of a proportional relationship as the slope of its line.
- Interpret the slope as a rate (miles per hour, dollars per pound, etc.) in real-world contexts.
- Compare two proportional relationships given in different representations (equation, table, graph, verbal description).

Big Idea

A **proportional relationship** between two quantities x and y has the form $y = kx$, where k is the **constant of proportionality** (also called the **unit rate**). When graphed, this relationship is always a **straight line that passes through the origin (0, 0)**. The constant k is exactly the **slope** of that line: it tells us how much y changes for every 1-unit increase in x . This single fact unifies arithmetic (ratios), algebra (equations), and geometry (lines) into one idea.

For example, if apples cost \$4 per pound, then the total cost y (in dollars) for x pounds is $y = 4x$. The unit rate is \$4/lb, the constant of proportionality is 4, and the slope of the line is 4. They are all the same number, because they all describe the same relationship in different languages: numerical (unit rate), symbolic (constant k), and geometric (slope m).

► Solution

Step 1 — Find the unit rate (cost per 1 pound). Pick any nonzero row. Using the row $x = 1$: the cost is \$8 for 1 lb, so the unit rate is \$8/lb. Verify with another row: $x = 4$ gives \$32, so $\$32 / 4 \text{ lb} = \$8/\text{lb}$.

Step 2 — Identify the constant of proportionality. $k = y/x = 8/1 = 8$ (consistent across all nonzero rows).

Step 3 — Write the equation. $y = kx \rightarrow y = 8x$.

Step 4 — Verify the slope from two points on the line. Take (1, 8) and (4, 32). Then $m = \Delta y / \Delta x = (32 - 8) / (4 - 1) = 24/3 = 8$.

Step 5 — Graph-check. The line through (0, 0) with slope 8 passes through (1, 8), (2, 16), (3, 24), (4, 32) — matching the table exactly. The unit rate (\$8/lb), the constant k (= 8), and the slope ($m = 8$) are all the same number.

Answer: $k = 8$; equation: $y = 8x$; slope $m = 8$ (= \$8 per pound, the unit rate).

Worked Example 2 — Reading Slope from a Proportional Graph

A proportional relationship is graphed below. Read two points from the graph and determine the slope, then write the equation and interpret it as a unit rate.

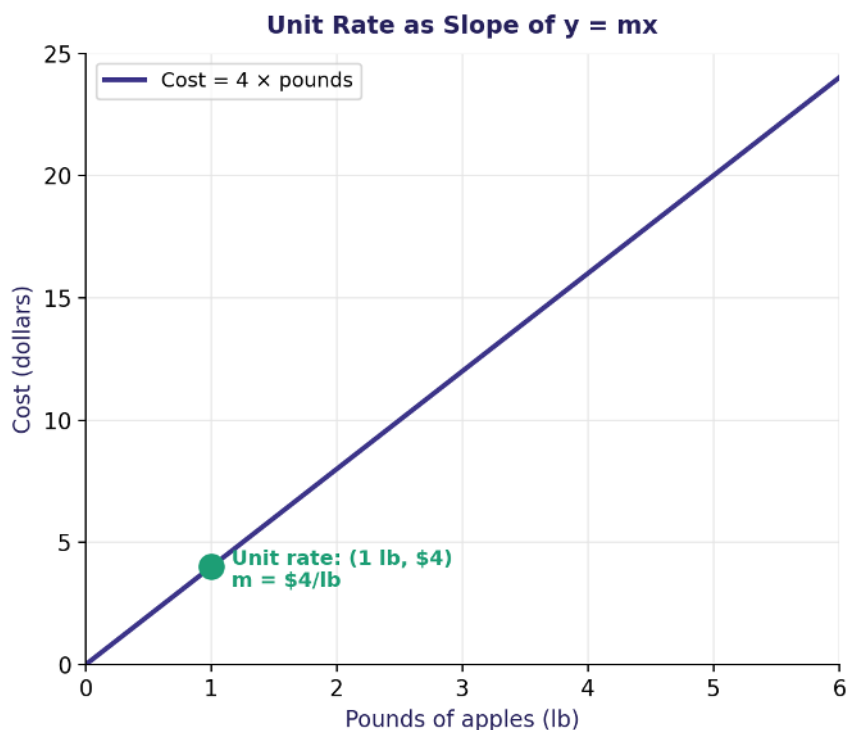


Figure 1.2 — Cost of apples vs. pounds. The point (1, 4) is the unit rate.

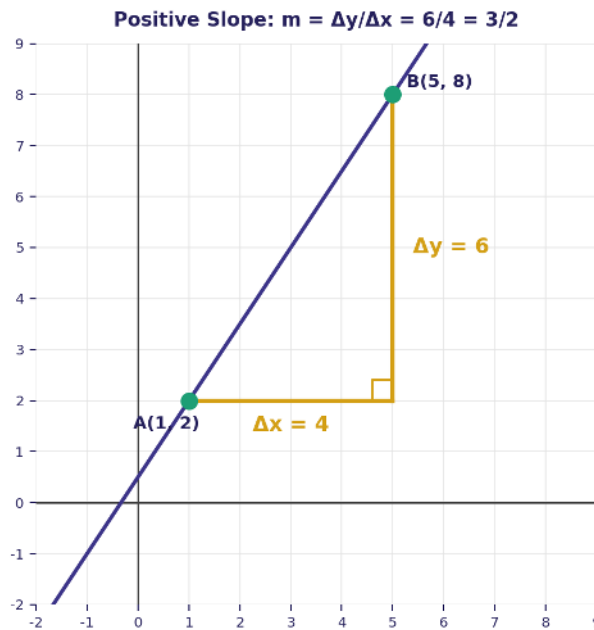


Figure 2.1 — Positive slope shown as $\Delta y / \Delta x$. The "rise" (Δy) is positive when going up.

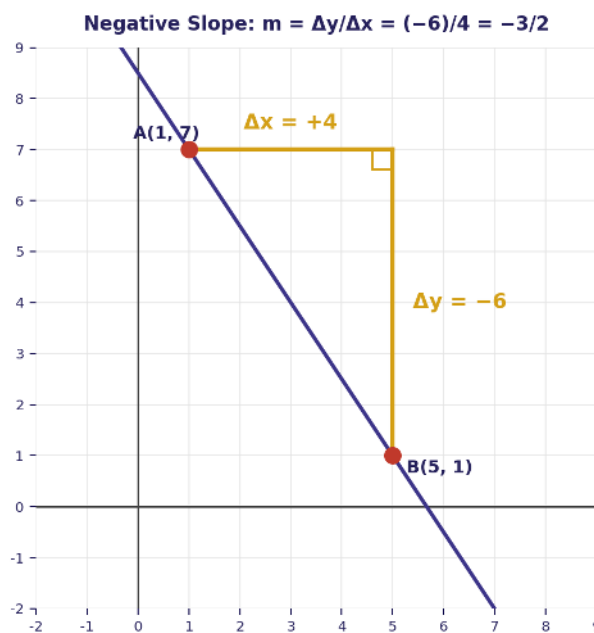


Figure 2.2 — Negative slope. Δy is negative (the line is going down as x increases), so the slope is negative.

Practice Set 2.A — 40-Problem Slope Computation Drill

CCSS 8.EE.B.6 | Problems from points, graphs, and tables. Full solutions follow the problem list.

1. A(1, 2), B(3, 6)	21. A(7, 7), B(-7, -7)
2. A(-1, 4), B(2, 1)	22. A(-4, 8), B(6, -7)
3. A(0, 0), B(5, 5)	23. A(0, 1), B(1, 0)
4. A(-2, -3), B(4, 6)	24. A(3, 9), B(-2, -6)
5. A(3, 7), B(3, -2)	25. A(1, 4), B(4, 13)
6. A(-4, 5), B(2, 5)	26. A line passes through A(2, 7) and B(2, -3). Classify its slope.
7. A(1, -2), B(4, -11)	27. A line passes through A(-4, 2) and B(5, 2). Classify its slope.
8. A(-5, -1), B(-1, 7)	28. A line passes through A(0, 0) and B(-3, 5). Is its slope positive, negative, zero, or undefined? Compute it.
9. A(0, 3), B(6, 0)	29. A line passes through A(-6, -4) and B(2, 4). Compute m and describe the line.
10. A(2, 2), B(8, 14)	30. A line passes through A(5, 0) and B(5, 9). Compute m and describe the line.
11. A(-3, 6), B(1, -2)	31. A line passes through A(0, 6) and B(8, 6). Compute m and describe the line.
12. A(4, 0), B(0, 6)	32. A line passes through A(-3, 5) and B(1, -3). Compute m and describe the line.
13. A(1, 1), B(2, 4)	33. A line on a graph passes through lattice points (0, 0) and (4, 6). Find m .
14. A(-6, -2), B(3, 4)	34. A line on a graph passes through (-2, 5) and (3, -5). Find m .
15. A(5, -3), B(-5, 7)	35. A line on a graph passes through (1, 4) and (5, 4). Find m .
16. A(1/2, 2), B(3/2, 6)	36. A line on a graph passes through (3, 1) and (3, 7). Find m .
17. A(-1/2, 3), B(5/2, -3)	37. Table: $x = 0, 2, 4, 6$; $y = 1, 5, 9, 13$. Find m .
18. A(2, 1/3), B(8, 7/3)	38. Table: $x = -2, 0, 2, 4$; $y = 8, 4, 0, -4$. Find m .
19. A(-3, 0), B(0, 4)	39. Table: $x = 1, 3, 5, 7$; $y = 2, 8, 14, 20$. Find m and verify with another pair.
20. A(0, -5), B(10, 0)	40. Table: $x = -3, -1, 1, 3$; $y = -5, 0, 5, 10$. Find m .

Solutions to 40-Problem Slope Drill

1. $\Delta y = 6 - 2 = 4$; $\Delta x = 3 - 1 = 2$ $m = 4/2 = 2 \rightarrow m = 2$

21. $\Delta y = -7 - 7 = -14$; $\Delta x = -7 - 7 = -14$ $m = (-14)/(-14) = 1 \rightarrow m = 1$

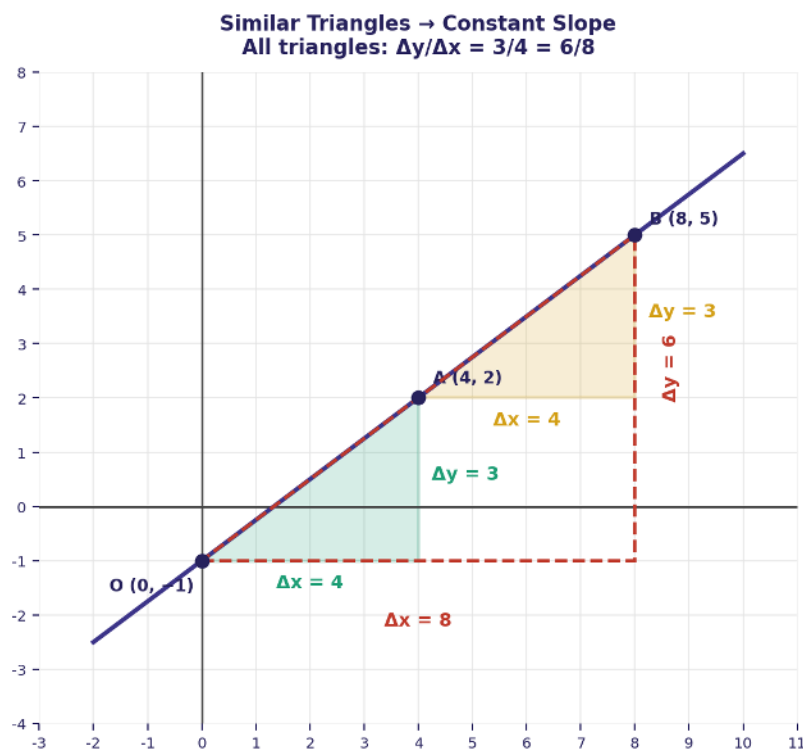


Figure 3.1 — Three similar slope triangles built on the same line. Each has $\Delta y/\Delta x = 3/4$ (small), $3/4$ (small), or $6/8 = 3/4$ (large). Same ratio = same slope.

KEY GEOMETRY — AA SIMILARITY CRITERION

AA Criterion: If two angles of one triangle equal two angles of another, the triangles are similar (their corresponding sides are proportional).

Similar Right Triangles (AA criterion): all angles equal → ratios equal
 $\triangle PQR$ (small)
 Legs 3, 4; hyp 5
 $\triangle P'Q'R'$ (large)
 Legs 6, 8; hyp 10

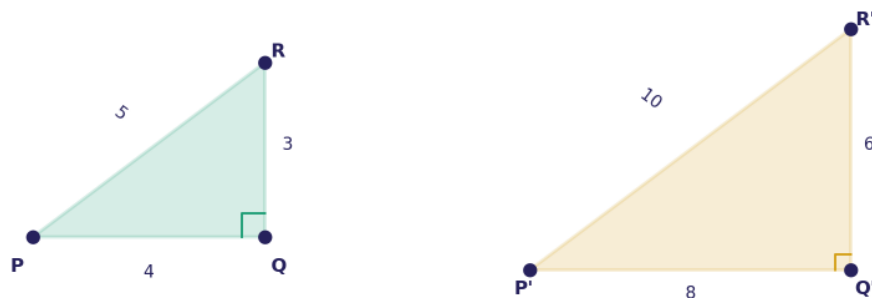


Figure 3.2 — Two right triangles with legs 3-4 and 6-8. Both have the same angle at the lower-left vertex (because the ratios $3/4$ and $6/8$ are equal), so by AA they are similar. Side ratios: $6/3 = 8/4 = 2$.

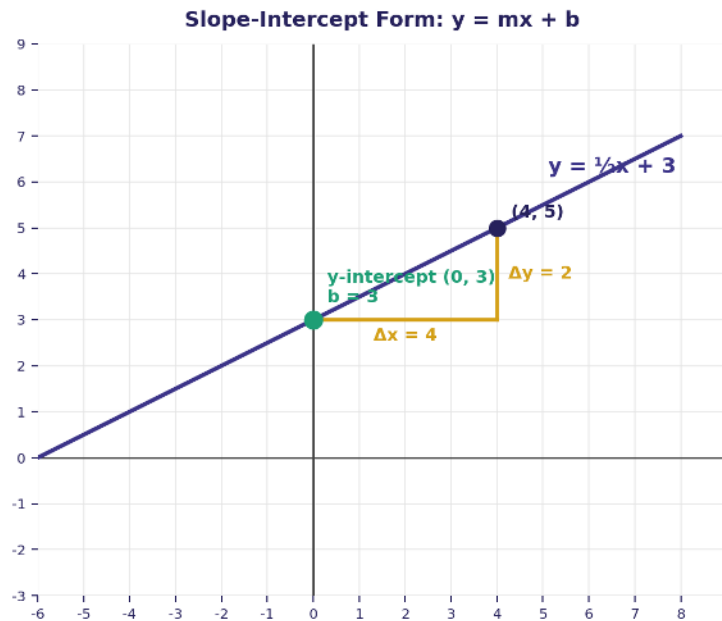


Figure 4.1 — Slope-intercept form $y = mx + b$. The slope m is the rise over run; b is where the line crosses the y -axis.

Derivation of $y = mx + b$

Let ℓ be a non-vertical line with slope m . Let $(0, b)$ be the y -intercept (the unique point on ℓ with $x = 0$; this exists because ℓ is non-vertical). Let (x, y) be any other point on ℓ . By the constant-slope property (Topic 3), the slope between $(0, b)$ and (x, y) equals m :

$$m = \Delta y / \Delta x = (y - b) / (x - 0) = (y - b) / x$$

$$mx = y - b \quad (\text{multiply both sides by } x)$$

$$y = mx + b \quad (\text{add } b \text{ to both sides})$$

This is the slope-intercept form. **If $b = 0$** (line passes through origin), it reduces to $y = mx$. This completes the derivation required by CCSS 8.EE.B.6.

Three Ways to Write the Equation of a Line

Given	Method	Example
Slope m and y-intercept b	Substitute directly: $y = mx + b$.	$m = 3, b = -2 \rightarrow y = 3x - 2$
Slope m and one point (x_1, y_1)	Plug into $y - y_1 = m(x - x_1)$, then solve for y .	$m = 2, (3, 5): y - 5 = 2(x - 3) \rightarrow y = 2x - 1$
Two points $(x_1, y_1), (x_2, y_2)$	(1) Compute $m = (y_2 - y_1) / (x_2 - x_1)$. (2) Use point-slope. (3) Solve for y .	$(1, 4), (3, 10): m = 3; y - 4 = 3(x - 1) \rightarrow y = 3x + 1$

Student Visual Guided Notes

Use these structured notes to capture the big ideas of the unit. Fill in every blank, sketch the diagrams in the boxes provided, and label every quantity. Completed notes become your study guide for the quiz and test.

1. Slope Formula Box

The slope m of a non-vertical line through points (x_1, y_1) and (x_2, y_2) is:

$$m = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

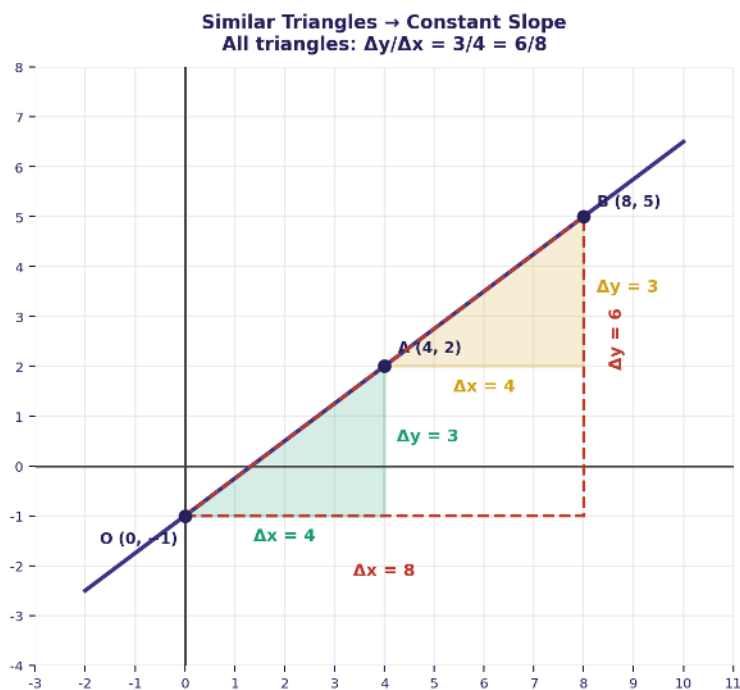
Δy is called the _____ (vertical change). Δx is called the _____ (horizontal change).

If $\Delta y > 0$ and $\Delta x > 0$, slope is _____ (positive / negative). If $\Delta y < 0$ and $\Delta x > 0$, slope is _____.

If $\Delta y = 0$, slope is _____. If $\Delta x = 0$, slope is _____.

2. Similar Triangles & Slope Diagram

Sketch the similar-triangles argument: draw a line, pick two pairs of points, build slope triangles, and label Δx and Δy on each. Then write the conclusion.



Comprehensive Unit Test

Time: 50 minutes | **Total:** 100 points | **Standards:** 8.EE.B.5, 8.EE.B.6, 8.F.A.2, 8.F.A.3

Instructions: Show all work for full credit. Slopes must be simplified fractions. Round decimals to 2 places only if a fraction cannot be written. Circle final answers.

Part I — Slope Computation (24 points, 4 each)

1. Find the slope of the line through A(1, 4), B(5, 16).
2. Find the slope of the line through A(-2, 7), B(3, -8).
3. Find the slope of the line through A(0, 0), B(7, 0).
4. Find the slope of the line through A(4, -1), B(4, 9).
5. Find the slope of the line through A(1/2, 3), B(5/2, 9).
6. Find the slope of the line through Table: x = 1, 3, 5, 7; y = 2, 8, 14, 20.

Part II — Equations of Lines (28 points, 7 each)

7. Write $y = mx + b$ for the line: Slope = 5, y-intercept = -3.
8. Write $y = mx + b$ for the line: Slope = $-2/3$, passes through (6, 1).
9. Write $y = mx + b$ for the line: Passes through (2, 3) and (6, 11).
10. Write $y = mx + b$ for the line: Passes through (-3, 5) and (0, -4).

Part III — Proportional Relationships & Comparison (24 points, 8 each)

11. A proportional relationship has $y = 24$ when $x = 6$. Write $y = mx$ and find y when $x = 15$.
12. Tank A: $y = 8x$ (gallons per minute). Tank B: 30 gal in 5 min. Which fills faster, and by how much?



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What's Inside the Complete Unit

Grade 8 Mathematics • 59 pages • 5 topic progressions • CCSS 8.EE.B.5, 8.EE.B.6, 8.F.A.2, 8.F.A.3

A complete sequence from proportional relationships and unit rate to slope, similar triangles, linear equations, and comparing representations.

01 PROPORTIONAL RELATIONSHIPS

Unit rate as slope; $y = mx$ through the origin; interpreting rates; comparing proportional relationships. Worked Examples 1-3, error analysis, exit ticket, and 12-problem Practice Set 1.A with solutions.

02 SLOPE OF A NON-VERTICAL LINE

Change in y over change in x ; positive, negative, zero, and undefined slope; graphs, tables, and constant rate of change. Worked Examples 4-8 and a 40-problem drill with solutions.

03 SIMILAR TRIANGLES & SLOPE

AA similarity and the argument that slope is constant on a non-vertical line. Includes labelled diagrams, Worked Examples 9-10, and a 15-problem reasoning set.

04 EQUATIONS OF LINES

Derivation of $y = mx + b$; equations from slope + intercept, slope + one point, and two points. Worked Examples 11-13 and a 25-problem Practice Set 4.A.

05 COMPARING REPRESENTATIONS

Translate among equation, table, graph, and verbal descriptions, then compare unit rates. Includes a representation translation map, Worked Examples 14-16, and a 20-problem Practice Set 5.A.

TEACHER + STUDENT SUPPORTS

Teacher notes; 10-day pacing guide; prerequisites; differentiation; key vocabulary; CCSS-M standards map; research-based interactive features; learning targets and big ideas; student visual guided notes; and an "I Can" checklist.

59

PAGES

5

TOPIC PROGRESSIONS

112TOPIC-SET PRACTICE
PROBLEMS**156**NUMBERED QUESTIONS
ACROSS PRACTICE + REVIEW +
QUIZ + TEST

Assessment & Review: Cumulative Unit Review (20 questions) • Unit Quiz (30 points) • Comprehensive Unit Test (100 points) • Full worked solutions / answer keys

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