

>> Comprehensive unit test with complete answer key

UM Printables - Premium Mathematics Resources

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TOPIC 1: ARITHMETIC SEQUENCES

An **arithmetic sequence** is a sequence where each term after the first is found by adding a constant value called the **common difference (d)** to the preceding term. The pattern of addition creates a linear relationship between the term number and the term value.

Arithmetic Sequence: 3, 7, 11, 15, 19, 23, ...



Identifying the Common Difference

The **common difference (d)** is the constant added to each term to get the next. To find d, subtract any term from the term that follows it:

$$d = a_2 - a_1 = a_3 - a_2 = a_{n+1} - a_n$$

NOTE: If $d > 0$, the sequence is increasing. If $d < 0$, the sequence is decreasing. If $d = 0$, every term is identical (constant sequence).

Example 1 Find the common difference of the arithmetic sequence: 5, 12, 19, 26, ...

Step 1: Subtract consecutive terms.

$$d = 12 - 5 = 7$$

$$d = 19 - 12 = 7$$

$$d = 26 - 19 = 7$$

The common difference is $d = 7$.

Example 2 Find the common difference of the arithmetic sequence: 100, 93, 86, 79, ...

Step 1: Subtract consecutive terms.

$$d = 93 - 100 = -7$$

$$d = 86 - 93 = -7$$

The common difference is $d = -7$. (Decreasing sequence)

The Explicit Formula

The **explicit formula** allows you to find any term a_n directly, given the first term a_1 and the common difference d.

EXPLICIT FORMULA FOR ARITHMETIC SEQUENCES

$$a_n = a_1 + (n - 1)d$$

TIP: The explicit formula is linear in form: $a_n = dn + (a_1 - d)$. Here, d is the slope (rate of change) and $(a_1 - d)$ is the y-intercept.

Example 3 Write the explicit formula for the sequence 3, 8, 13, 18, ... and find the 20th term.

Step 1: Identify a_1 and d .

$$a_1 = 3, d = 8 - 3 = 5$$

Step 2: Substitute into the formula $a_n = a_1 + (n - 1)d$.

$$a_n = 3 + (n - 1)5$$

$$a_n = 3 + 5n - 5$$

$$a_n = 5n - 2$$

Step 3: Find the 20th term ($n = 20$).

$$a_{20} = 5(20) - 2 = 100 - 2 = 98$$

The Recursive Formula

A **recursive formula** defines each term based on the previous term. It requires the first term and the rule to get from one term to the next.

RECURSIVE FORMULA FOR ARITHMETIC SEQUENCES

$$a_1 = \dots$$

$$a_n = a_{n-1} + d, \text{ for } n > 1$$

Explicit Formula	Recursive Formula
Finds any term directly. $a_n = a_1 + (n - 1)d$ Independent of other terms.	Requires the previous term. $a_1 = \text{value}, a_n = a_{n-1} + d$ Must compute all prior terms.

Example 4 Write the recursive formula for the sequence 50, 44, 38, 32, ...

Step 1: Identify a_1 .

$$a_1 = 50$$

Step 2: Identify the common difference d .

$$d = 44 - 50 = -6$$

Step 3: Write the recursive formula.

$$a_1 = 50$$

$$a_n = a_{n-1} - 6, \text{ for } n > 1$$

Finding Terms & Term Numbers

You can use the explicit formula to find a specific term number (n) when given the term value, or to find the term value when given n .

Example 5 Which term number of the sequence 2, 9, 16, 23, ... has a value of 107?

Step 1: Identify a_1 and d .

$$a_1 = 2, d = 9 - 2 = 7$$

Step 2: Set up the explicit formula with $a_n = 107$.

$$107 = 2 + (n - 1)7$$

Step 3: Solve for n .

$$105 = 7(n - 1)$$

$$15 = n - 1$$

$$n = 16$$

The value 107 is the 16th term.

Arithmetic Means

An **arithmetic mean** is a term inserted between two non-consecutive terms of an arithmetic sequence. If one term is inserted, it is the arithmetic mean. If multiple terms are inserted, they are equally spaced between the given terms.

**KEY
POINT:**

To find arithmetic means between a_1 and a_n , determine the total number of terms (including the means), then find d using the explicit formula, and finally calculate each mean.

Example 6 Insert three arithmetic means between 5 and 25.

Step 1: Understand the problem. We have $a_1 = 5$ and $a_5 = 25$.

(The three means will be a_2 , a_3 , and a_4 .)

Step 2: Use the explicit formula to find d .

$$a_n = a_1 + (n - 1)d$$

$$25 = 5 + (5 - 1)d$$

$$20 = 4d$$

$$d = 5$$

Step 3: Find the means by adding d successively.

$$a_2 = 5 + 5 = 10$$

$$a_3 = 10 + 5 = 15$$

$$a_4 = 15 + 5 = 20$$

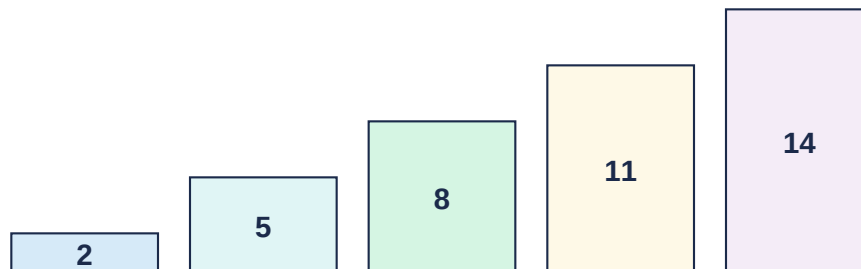
The three arithmetic means are 10, 15, and 20.

TOPIC 2: ARITHMETIC SERIES & SIGMA NOTATION

Arithmetic Series & The Partial Sum Formula

An **arithmetic series** is the sum of the terms of an arithmetic sequence. While we could add the terms one by one, the **partial sum formula** provides a fast way to find the sum of the first n terms.

$$S_5 = 2 + 5 + 8 + 11 + 14 = 40$$



$$S_n = n/2 * (a_1 + a_n) = 5/2 * (2 + 14) = 40$$

PARTIAL SUM FORMULA FOR ARITHMETIC SERIES

$$S_n = n/2 (a_1 + a_n)$$

OR

$$S_n = n/2 [2a_1 + (n - 1)d]$$

TIP: Use $S_n = n/2 (a_1 + a_n)$ when you know the first and last terms. Use $S_n = n/2 [2a_1 + (n - 1)d]$ when you know the common difference but not the last term.

Example 1 Find the sum of the first 25 terms of the arithmetic sequence: 3, 7, 11, 15, ...

Step 1: Identify knowns.

$$a_1 = 3, d = 4, n = 25$$

Step 2: Find the 25th term, a_{25} .

$$a_{25} = 3 + (25 - 1)4 = 3 + 24(4) = 3 + 96 = 99$$

Step 3: Apply the sum formula $S_n = n/2 (a_1 + a_n)$.

$$S_{25} = 25/2 (3 + 99)$$

$$S_{25} = 25/2 (102)$$

$$S_{25} = 25(51) = 1275$$

Example 2 Find the sum: $18 + 14 + 10 + \dots + (-46)$

Step 1: Identify knowns.

$$a_1 = 18, a_n = -46, d = -4$$

Step 2: Find the number of terms, n .

TOPIC REVIEW: CUMULATIVE UNIT REVIEW

Assess your understanding of all five topics. Complete these problems without looking at your notes. Check your work using the answer key provided at the end of the resource.

Arithmetic Sequences & Series

1. Write the explicit formula for the sequence: 15, 8, 1, -6, ...
2. Write the recursive formula for the sequence: 100, 92, 84, 76, ...
3. Find the 30th term of the arithmetic sequence where $a_1 = -5$ and $d = 4$.
4. Insert two arithmetic means between 4 and 22.
5. Find the sum of the first 50 terms: $3 + 7 + 11 + 15 + \dots$
6. Evaluate: $\sum_{k=1}^{15} (4k - 2)$

Geometric Sequences & Series

1. Write the explicit formula for the sequence: 5, -15, 45, -135, ...
2. Find the 8th term of the geometric sequence: 2, 6, 18, 54, ...
3. Find the sum of the first 7 terms of the geometric series: $3 + 6 + 12 + \dots$
4. Determine if the infinite series converges: $40 - 20 + 10 - 5 + \dots$. If so, find the sum.
5. Evaluate: $\sum_{k=1}^5 4(0.5)^{k-1}$

Sigma Notation & The Binomial Theorem

1. Write the series in sigma notation: $1 + 8 + 27 + 64 + 125$
2. Evaluate: $\sum_{k=1}^{\text{infinity}} 6(2/3)^{k-1}$
3. Expand completely: $(x + 2)^4$
4. Find the 4th term in the expansion of $(2x - y)^6$
5. Find the term containing x^3 in the expansion of $(x + 1)^5$

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