



PRINTABLES
PREMIUM EDUCATIONAL RESOURCES

SECONDARY MATHEMATICS • PROJECT-BASED LEARNING UNIT

Right Triangle Trigonometry

Surveying & Indirect Measurement

A 3–5 day, hands-on Project-Based Learning unit that rebuilds conceptual understanding of right triangle trigonometry through authentic indirect measurement. Designed for **RTI Tier 2** intervention, this resource pairs concrete, real-world stations with explicit vocabulary instruction, scaffolded **I do / We do / You do** procedure cards, an error-correction protocol, and progress-monitoring exit tickets — including a Tier 3 modified version.

GRADE BAND	INTERVENTION	DURATION	STATIONS
9–11	RTI Tier 2	3–5 days	4

Clinometer • Similar Triangles • Shadow Method • Tangent Ratio • Angle of Elevation • Inclinator • Slope • SOH CAH TOA

Table of Contents

Table of Contents	2
1. Teacher Overview & Implementation Guide	3
2. The RTI Tier 2 Framework & Scaffolding Architecture	6
3. Vocabulary Pre-Teaching with Visual Anchors	8
4. Concept Refresher — The Trigonometric Ratios	11
5. Station 1 — Tree Height with a Clinometer	13
6. Station 2 — Building Height via Shadows	17
7. Station 3 — River Width	20
8. Station 4 — Slope of a Ramp or Staircase	23
9. Detailed Solutions — Exit Tickets & Worked Examples	26
10. Tier 3 Modified Version — Reduced Ratios, Calculator-Supported	28
11. Error-Correction Protocol	30
12. Student Reflection	32
13. Real-World Trigonometry Careers	34
A Note from UM Printables	36

1. Teacher Overview & Implementation Guide

Why This Unit Exists

Many students who receive initial trigonometry instruction through abstract symbolic manipulation fail to build a durable mental model of what the trigonometric ratios *actually describe*. They memorise SOH CAH TOA for a quiz, then cannot decide which ratio to use when a triangle is rotated, embedded in a word problem, or oriented vertically as a real object. This unit rebuilds that missing conceptual foundation through the most powerful corrective available to Tier 2 intervention: **concrete, authentic, indirect measurement**. When a student sights a clinometer at the top of a real tree and computes its height with the tangent ratio, the abstract suddenly becomes physical. The ratio is no longer a formula to recall — it is a tool that produced a verifiable result.

The unit is built around four measurement stations drawn from the historical practice of **surveying**, the discipline that gave birth to trigonometry. Each station asks students to determine an inaccessible quantity (a height, a width, a slope) using only what they can directly measure plus one trigonometric relationship. This is precisely the intellectual problem that motivated the development of trigonometry in the first place, which gives the unit genuine disciplinary authenticity rather than manufactured relevance.

Unit at a Glance

The recommended pacing below spreads the four stations across three to five instructional blocks. Day 1 establishes vocabulary and the conceptual refresher; Days 2–4 run the stations (one per day, or two per day on a compressed five-day schedule with the vocabulary and stations merged); Day 5 is reserved for consolidation, reflection, and the careers connection. Each station is self-contained, so teachers may reorder them to suit facility, weather, and equipment availability.

Day	Focus	Core Activity	Assessment
1	Vocabulary & Concept Refresher	Explicit pre-teaching of triangle anatomy and ratios; SOH CAH TOA; worked examples	Diagnostic check; vocabulary card check
2	Station 1 — Tree Height (Clinometer)	Build clinometer; measure a tree or pole; tangent ratio	Exit Ticket 1
3	Station 2 — Building Height (Shadows)	Shadow & similar-triangle method; verify with tangent	Exit Ticket 2
4	Stations 3 & 4 — River Width & Ramp Slope	Congruent-triangle sighting; inclinometer app & tangent	Exit Tickets 3 & 4
5	Consolidation & Reflection	Solutions review, error-correction, reflection, careers	Reflection rubric; unit mastery check

Learning Objectives

By the end of this unit, students will be able to:

3. Vocabulary Pre-Teaching with Visual Anchors

Vocabulary instruction precedes every other activity in this unit. The five terms below are the load-bearing vocabulary of right-triangle trigonometry; a student who cannot fluently identify the opposite and adjacent sides relative to a given angle cannot apply any trigonometric ratio. Teach these terms with the visual anchors provided, and require students to use the precise term (not “the long side” or “the slanted part”) throughout the stations.

The Reference Triangle

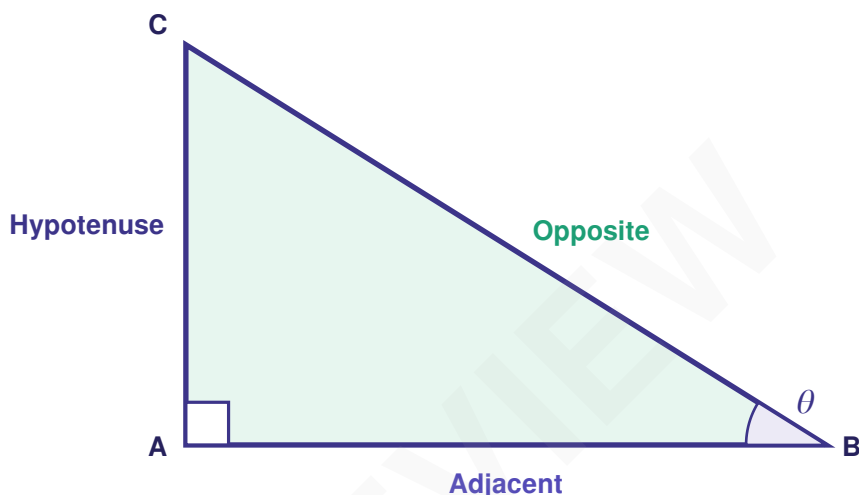


Figure 3.1 — The right triangle, labelled relative to reference angle θ at vertex B.

Vocabulary Cards

Print or project the following vocabulary cards. Each card gives a student-friendly definition and a visual anchor. Have students build a foldable or flash-card set by copying each card; the act of reproducing the diagram is itself an encoding activity.

Hypotenuse

The side **opposite the right angle**. It is always the **longest** side of a right triangle. It never touches the 90° corner.

Visual anchor: In Figure 3.1, side AC. It is the side that does *not* form the right angle at A.

Opposite (leg)

The leg **across from** the reference angle θ . It does **not** form the reference angle. Its length changes if you choose a different reference angle.

Visual anchor: In Figure 3.1, with θ at B, the opposite side is BC (the rising side).

4. Concept Refresher — The Trigonometric Ratios

Before students pick up a clinometer, they need a crisp, durable understanding of the three trigonometric ratios and why the **tangent** ratio is the workhorse of indirect measurement. This refresher is deliberately concrete: it rebuilds the ratio from a comparison of sides rather than declaring it by fiat.

SOH CAH TOA — The Three Ratios

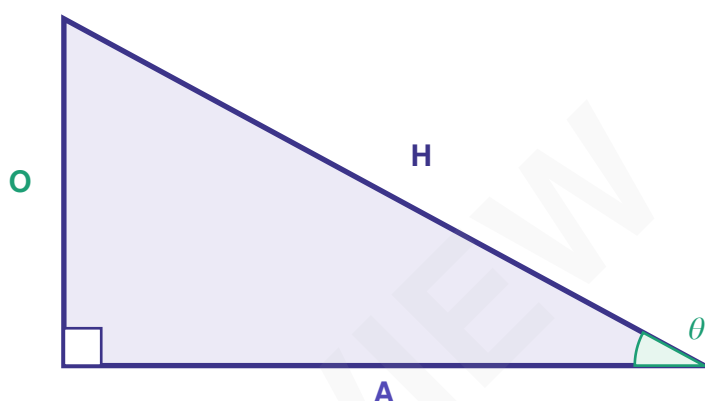


Figure 4.1 — Reference triangle for the three primary trigonometric ratios.

Ratio	Mnemonic c	Formula (relative to θ)	Use it when you know...	and want to find...
sine	SOH	$\sin(\theta) = O / H$	the angle and the hypotenuse	the opposite leg
cosine	CAH	$\cos(\theta) = A / H$	the angle and the hypotenuse	the adjacent leg
tangent	TOA	$\tan(\theta) = O / A$	the angle and one leg	the other leg

Why Tangent Dominates Indirect Measurement

In a field-measurement context, the two quantities you can *directly* measure with a tape are usually two legs of a right triangle: a horizontal distance along the ground (the adjacent) and a vertical height (the opposite). You typically cannot directly measure the slanted hypotenuse — that is the whole point of indirect measurement. Because the tangent ratio relates the two **legs**, it is the ratio you reach for whenever a tape measure (not a sonic rangefinder) is your tool. Sine and cosine come into play when a slanted distance *is* measurable, which is rarer in school settings.

Anchor Concept

The big idea, in one sentence: Whenever you can measure a horizontal distance and an angle, the tangent ratio gives you the vertical height you cannot reach — and vice versa. Every station in this unit is a variation on that single idea.

1

Station 1 — Tree Height with a Clinometer

Materials: protractor, straw, string, washer, tape measure, calculator

Station Goal

Determine the height of a tall object (a tree, flagpole, or light pole) that you cannot directly measure, by sighting the angle to its top with a clinometer and applying the tangent ratio.

Part A — Build Your Clinometer

A clinometer is a device that measures the angle between your line of sight and the horizontal. The version below uses a protractor as the angle scale, a straw as the sight tube, and a weighted string as a plumb line that always hangs vertically. Build one per pair before going outside.

Clinometer parts

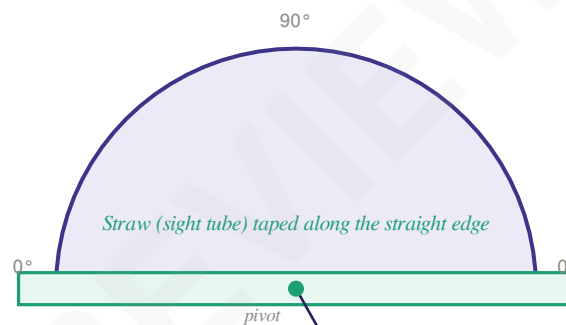


Figure 5.1 — Anatomy of a homemade clinometer.

Washer (plumb bob weight)

Step	Action
1	Take a protractor and lay it flat, curved side down, straight edge along the top.
2	Tape a drinking straw along the straight (flat) edge so it runs the full width. This is your sight tube.
3	Punch a small hole at the centre of the straight edge (the protractor's origin point).
4	Thread ~30 cm of string through the hole and knot it so it cannot pull back through.
5	Tie a washer or small bolt to the free end of the string. This is your plumb bob — it will always hang straight down.
6	Calibrate: hold the clinometer so the straw is perfectly horizontal (sight a point level with your eye). The string should hang over the 90° mark. If your protractor is labelled 0–180, the horizontal reading is 90°; the angle of elevation = 90° – (string reading).

Pro Tip — Reading the Clinometer

Reading the angle: When you sight up through the straw, the string swings to a new position. Read the protractor value where the string crosses the scale. **Angle of elevation = $|90^\circ - \text{reading}|$.** For example, if the string reads 58° , your angle of elevation is $90^\circ - 58^\circ = 32^\circ$. Many teachers prefer to re-label the protractor in advance so the reading is the elevation angle directly.

Part B — The Measurement Model

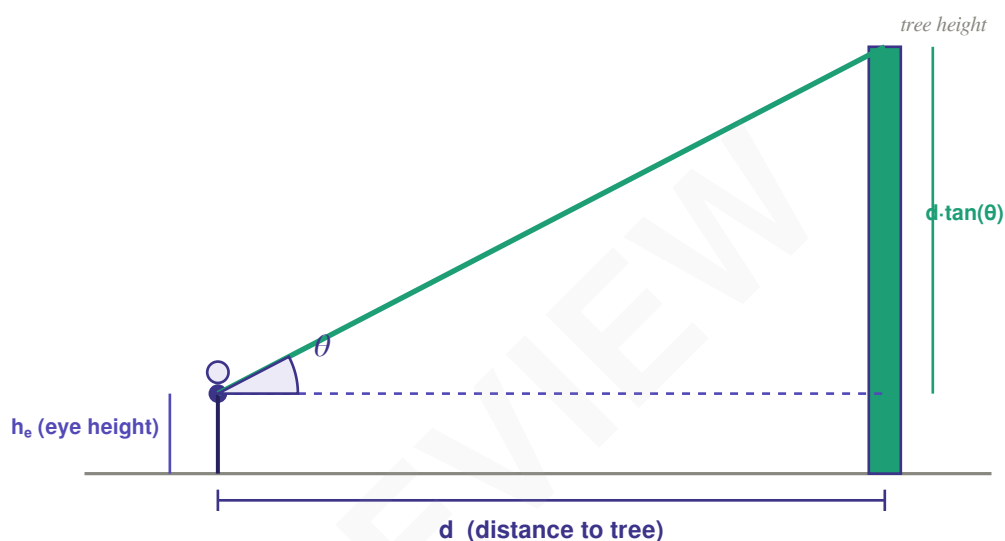


Figure 5.2 — The geometry of a clinometer height measurement. The total tree height = eye height + (distance $\times \tan \theta$).

The Height Formula

Master equation:

$$\text{height} = h_{\text{eye}} + d \times \tan(\theta)$$

where h_{eye} = height of the observer's eye above the ground, d = horizontal distance from observer to the base of the object, and θ = angle of elevation to the top.

Part C — Procedure Card (I do / We do / You do)

1

I DO (Teacher models)

The teacher selects a pole, stands at a marked 10 m distance, sights the top through the clinometer, reads the string, computes the elevation angle, and calculates the height aloud at each step. Students observe only.

2

WE DO (Guided practice)

The class moves to a second object. A student volunteer holds the clinometer; the class chorally supplies each step as the teacher prompts: "What do we measure first? ... Which ratio? • What do we substitute?" The teacher records data on the board and all students copy it into their data sheet.

2

Station 2 — Building Height via Shadows

Similar-triangle method • Materials: tape measure, metre stick, sunny day

Station Goal

Determine the height of a tall building (or wall, or pole) by measuring shadows and using the geometric fact that, at a given moment, the sun's rays are parallel — so the building and a short reference object cast **similar triangles**. Then verify the result with the tangent ratio.

Part A — The Similar-Triangle Principle

Similar triangles: $h_1 / s_1 = h_2 / s_2 = \tan(\theta)$

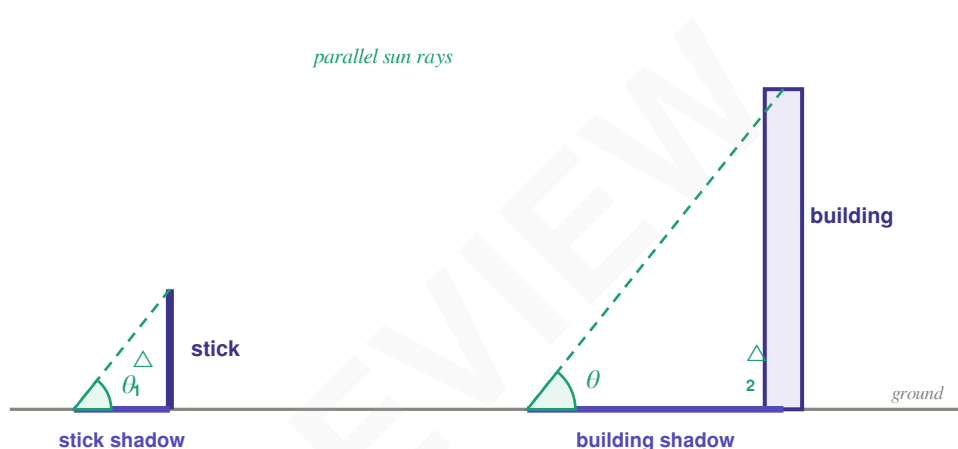


Figure 6.1 — Parallel sun rays produce similar triangles. The ratio height/shadow is the same for every object at the same instant.

Because the sun is very far away, its rays arrive essentially parallel. The triangle formed by a vertical object, its shadow, and the sun ray to its top is therefore similar (by AA — a right angle and a common sun angle) to the triangle formed by any other vertical object and its shadow at the same moment. Corresponding sides of similar triangles are proportional, giving the master equation below.

The Shadow Equation

Master equation (shadow method):

$$\frac{H_{\text{building}}}{\text{shadow}_{\text{building}}} = \frac{h_{\text{stick}}}{\text{shadow}_{\text{stick}}}$$

Equivalently: $H_{\text{building}} = \text{shadow}_{\text{building}} \times \left(\frac{h_{\text{stick}}}{\text{shadow}_{\text{stick}}} \right)$

The ratio $h_{\text{stick}} / \text{shadow}_{\text{stick}}$ is itself $\tan(\theta)$ where θ is the sun's elevation angle — which is why the shadow method and the clinometer method must agree.

Part B — Procedure Card (I do / We do / You do)

3

Station 3 — River Width (Tape Measure + Straw)

Set up in a hallway or field — no actual river required!

Station Goal

Determine an inaccessible horizontal distance — the width of a ‘river’ (a taped-off strip of floor or grass) — using **only a tape measure and a drinking straw**, with no angle-measuring device. The method relies on constructing a pair of congruent triangles by sight.

Part A — The Method

Congruent triangles ($\triangle ABC \cong \triangle DEC$): $AB = DE$

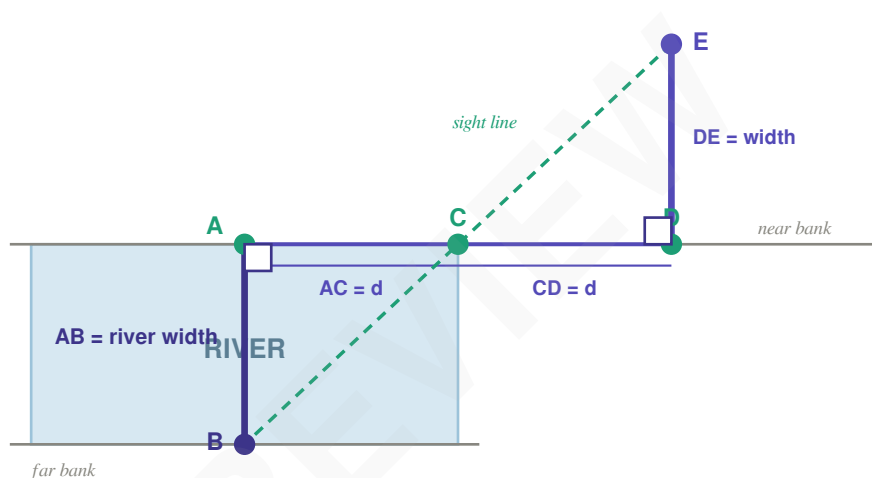


Figure 7.1 — The congruent-triangle sighting method. Because $AC = CD$ and the angles at C are vertically opposite (equal), the river width AB equals the measured distance DE .

Place a landmark B on the far side of the ‘river’ (a chair, cone, or taped X). Stand directly opposite it on the near bank at point A . Walk along the near bank a measured distance to point C , then continue an equal distance to point D (so C is the midpoint of AD). At D , turn 90° away from the river and walk, sighting backward through the straw, until the straw aligns C and B (you see B directly over C). Mark this point E . Measure DE with the tape. By congruent triangles, $DE = AB =$ the river width.

The River-Width Proof

Why it works (the proof in three lines):

- $\triangle ABC$ has a right angle at A (AB is across the river, AC along the bank).
- $\triangle DEC$ has a right angle at D (DE is perpendicular to the bank, DC along it).
- $\angle ACB = \angle DCE$ (vertically opposite angles) and $AC = DC$ (we made them equal), so by ASA the triangles are congruent $\rightarrow AB = DE$.

Master equation: river width $AB = DE$ (the distance you measured on land).

Part B — Procedure Card (I do / We do / You do)

4

Station 4 — Slope of a Ramp or Staircase

Materials: tape measure, inclinometer app (phone), calculator

Station Goal

Measure the slope of a real ramp (or staircase) two ways — as a rise/run ratio using a tape measure, and as an angle using an inclinometer app — then confirm that the two agree through the tangent ratio. Finally, judge the ramp against the accessibility standard.

Part A — Slope as a Tangent

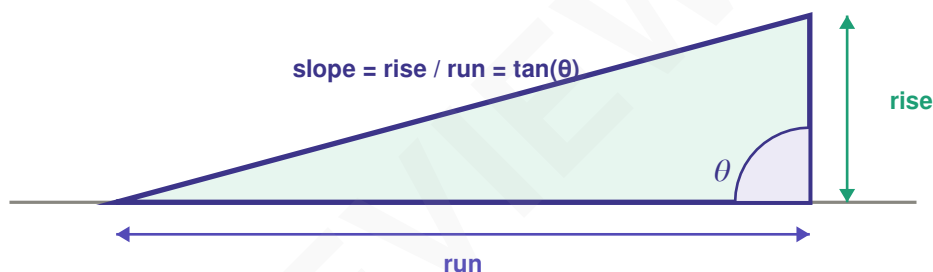


Figure 8.1 — A ramp is a right triangle in disguise. Its slope (rise / run) is exactly $\tan(\theta)$, where θ is the angle of inclination.

Slope is the single most common application of the tangent ratio in adult life. Every ramp, road, staircase, roof, and wheelchair grade is described by a tangent. Civil engineers express slope three equivalent ways: as a **ratio** (1 : 12), as a **percentage** (8.3%), and as an **angle** (4.8°). All three are the same number wearing different clothes.

Slope = $\tan(\theta)$

Three equivalent slope languages:

- **Ratio:** rise : run (e.g. 1 : 12 means 1 unit up for every 12 across)
- **Decimal / percent:** $\text{rise} \div \text{run}$ ($0.0833 = 8.33\%$)
- **Angle:** $\theta = \arctan(\text{rise} \div \text{run})$

ADA accessibility standard: a wheelchair ramp must have slope $\leq 1 : 12$ ($\leq 8.33\%$, $\theta \leq 4.76^\circ$).

Part B — Procedure Card (I do / We do / You do)

1

I DO (Teacher models)

The teacher selects a ramp, measures its run and rise with a tape, computes slope as a ratio and percent, then lays a phone with the inclinometer app flat on the ramp surface and reads the angle, confirming $\tan(\theta)$ matches rise/run.

9. Detailed Solutions — Exit Tickets & Worked Examples

This section consolidates every exit-ticket solution with full working. Distribute to students after they have attempted the tickets independently; the comparison of a student's own working against the model solution is itself a powerful learning act. Each solution names the ratio used, shows the substitution, performs the computation, and ends with a sanity check — the same four-move structure students should internalise.

Exit Ticket 1 — Clinometer / Tree Height

Given: $d = 15$ m, clinometer reading $= 52^\circ$, $h_{\text{eye}} = 1.6$ m.

Solution — Exit Ticket 1

Q1 — Angle of elevation. The string hangs at 90° when the straw is horizontal. When it reads 52° , the straw has tilted up by $90^\circ - 52^\circ = 38^\circ$. So $\theta = 38^\circ$.

Q2 — Equation for the height above eye. The opposite side (height above eye) and adjacent side (15 m) give $\tan(38^\circ) = \text{opposite} / 15$, so $\text{opposite} = 15 \times \tan(38^\circ)$.

Q3 — Compute height above eye. $\tan(38^\circ) \approx 0.7813$. $\text{opposite} = 15 \times 0.7813 = 11.7195 \approx \mathbf{11.72}$ m.

Q4 — Total tree height. $H = h_{\text{eye}} + \text{opposite} = 1.6 + 11.72 = \mathbf{13.32}$ m (about 13.3 m).

Sanity check. A 13 m tree is a mid-sized shade tree; a 38° sight angle at 15 m gives a height above eye of just under 12 m, which is arithmetically consistent ($15 \times 0.78 \approx 11.7$).

Exit Ticket 2 — Shadows / Building Height

Given: stick height = 1.5 m, stick shadow = 2.0 m, building shadow = 24 m.

Solution — Exit Ticket 2

Q1 — Ratio. height / shadow $= 1.5 / 2.0 = \mathbf{0.75}$. (This number is $\tan(\theta)$ where θ is the sun's elevation.)

Q2 — Proportion. By similar triangles, $H / 24 = 1.5 / 2.0$.

Q3 — Solve. $H = 24 \times (1.5 / 2.0) = 24 \times 0.75 = \mathbf{18.0}$ m.

Q4 — Sun angle. $\theta = \arctan(0.75)$. Using a calculator, $\arctan(0.75) \approx 36.87^\circ$, so $\theta \approx \mathbf{36.9^\circ}$.

Sanity check. The building is four storeys (18 m) and casts a shadow 24 m long — a shadow longer than the building, exactly what a 37° sun produces (shadow = height / $\tan(37^\circ) = 18 / 0.75 = 24$). ✓

Exit Ticket 3 — River Width

Given: $AC = CD = 10$ m, DE (trial 1) = 16.8 m, DE (trial 2) = 17.0 m.

Right Triangle Trigonometry PBL

Surveying & Indirect Measurement • RTI Tier 2

A 36-page PBL unit built around four real-world measurement stations and structured intervention supports.

36

PAGES

4

STATIONS

4

EXIT TICKETS

13

SECTIONS

COMPLETE TABLE OF CONTENTS

- | | |
|--|--|
| <p>1 Teacher Overview & Implementation Guide
Objectives, recommended 3–5 day pacing, materials, and assessment architecture.</p> <p>2 RTI Tier 2 Framework & Scaffolding
Targeted intervention framework, three design pillars, and gradual release.</p> <p>3 Vocabulary Pre-Teaching with Visual Anchors
Triangle vocabulary cards plus angle-of-elevation/depression anchors and self-check.</p> <p>4 Concept Refresher — Trigonometric Ratios
SOH CAH TOA, reference-triangle visuals, tangent focus, and worked examples.</p> <p>5 Station 1 — Tree Height with a Clinometer
Build, calibrate, measure, record, solve, and check using tangent.</p> <p>6 Station 2 — Building Height via Shadows
Similar-triangle shadow method with tangent verification.</p> <p>7 Station 3 — River Width
Tape-and-straw sighting method with a congruent-triangle proof.</p> | <p>8 Station 4 — Slope of a Ramp or Staircase
Measure rise/run and angle, connect slope to $\tan(\theta)$, and compare with the cited accessibility standard.</p> <p>9 Detailed Solutions — Exit Tickets & Worked Examples
Step-by-step solutions for all four exit tickets plus calculator-error reference.</p> <p>10 Tier 3 Modified Version — Reduced Ratios, Calculator-Supported
Sin/cos-only pathway, simplified station set, worked examples, and a calculator frame card.</p> <p>11 Error-Correction Protocol
Four-step correction routine plus six common trigonometry errors and targeted fixes.</p> <p>12 Student Reflection
Eight reflection prompts plus a next-step goal-setting page.</p> <p>13 Real-World Trigonometry Careers
Eight career profiles, career-connection writing prompt, and a history connection.</p> |
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THE FULL RESOURCE ALSO INCLUDES

Student data-recording materials, station procedure cards, worked examples, four exit tickets, detailed solutions, a calculator-error quick reference, the Tier 3 modified pathway, error-correction routines, reflection, and career-connection activities.