

Rational Exponents, Radicals & Complex Numbers

Complete Unit — Grades 10–11 (Refined Edition)

Guided Notes • Practice Drills • Graphing Activities • Quizzes • Unit Test •
Detailed Solutions

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Topic 1: Rational Exponents

Guided Notes

Definition: Rational Exponent Notation

If n is a positive integer greater than 1, then $a^{1/n} = \sqrt[n]{a}$. More generally, $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$. The denominator of the rational exponent becomes the **index** of the radical. The numerator of the rational exponent becomes the **exponent** of the radicand. When the base is negative, the denominator must be odd for the result to be a real number.

Example 1. Convert from rational exponent to radical form: $27^{2/3}$

- **Identify.** The exponent is $2/3$ — numerator 2, denominator 3.
- **Apply rule.** Denominator 3 becomes the index (cube root); numerator 2 becomes the power on the radicand.
- **Convert.** $27^{2/3} = (\sqrt[3]{27})^2$
- **Evaluate.** $\sqrt[3]{27} = 3$, so $(3)^2 = 9$

✓ **Final Answer: 9**

Example 2. Convert from radical to rational exponent form: $\sqrt[4]{16^3}$

- **Identify.** Index of radical = 4; exponent on radicand = 3.
- **Apply rule.** Index becomes denominator (4); exponent on radicand becomes numerator (3).
- **Convert.** $\sqrt[4]{16^3} = 16^{3/4}$
- **Evaluate (optional).** $16^{1/4} = 2$, so $2^3 = 8$

✓ **Final Answer: 8**

Properties of Rational Exponents

The same properties that apply to integer exponents also apply to rational exponents.

Product Rule: $a^{m/n} \cdot a^{p/q} = a^{(mq+np)/nq}$

Quotient Rule: $a^{m/n} \div a^{p/q} = a^{(mq-np)/nq}$

Power Rule: $(a^{m/n})^{p/q} = a^{mp/nq}$

Negative Exponent: $a^{-m/n} = 1 / a^{m/n}$

Zero Exponent: $a^0 = 1$ (for $a \neq 0$)

Key shortcut: For the Product and Quotient Rules, when the denominators are already equal you can simply add or subtract the numerators.

Example 3. Simplify using exponent rules: $(x^{2/3})(x^{4/3})$

- **Identify.** Same base x ; both exponents have denominator 3.
- **Apply Product Rule.** Add the exponents: $2/3 + 4/3 = 6/3 = 2$.
- **Simplify.** $x^{6/3} = x^2$

✓ **Final Answer:** x^2

Solving Equations with Rational Exponents

To solve an equation of the form $x^{m/n} = k$, raise both sides to the reciprocal power n/m . Always check for extraneous solutions, especially when the denominator (index) is even, as even roots of negative numbers are not real. After solving, substitute your answer back into the original equation to verify it is valid.

Example 4. Solve: $x^{3/2} = 64$

- **Identify.** Exponent $3/2$; reciprocal is $2/3$.
- **Apply.** Raise both sides to the $2/3$ power: $(x^{3/2})^{2/3} = 64^{2/3}$
- **Simplify left side.** $(3/2)(2/3) = 1$, so left side becomes x .
- **Evaluate right side.** $64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16$.
- **Verify.** $16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$. ✓

✓ **Final Answer:** $x = 16$

■ Watch Out! Extraneous Solutions

When solving equations where the variable has an even denominator in the exponent (e.g., $x^{1/2}$), squaring both sides can introduce **extraneous solutions** — answers that emerge from the algebra but do not satisfy the original equation. **Always substitute your final answer back into the original equation** to confirm it works. If a principal square root equals a negative number, that solution is extraneous and must be discarded.

Topic 1: Rational Exponents — Practice Set

Evaluate or simplify. Assume all variables represent positive real numbers. Show all work and check your solutions to equations.

1.

$$32^{1/5}$$

2.

$$27^{2/3}$$

3.

$$16^{3/4}$$

4.

$$(-8)^{2/3}$$

5.

$$81^{-1/2}$$

6.

$$100^{3/2}$$

7.

$$64^{2/3}$$

8.

$$(-125)^{1/3}$$

9.

$$9^{-3/2}$$

10.

Write $b^{4/5}$ in radical form.

11.

Write $\sqrt[6]{c^5}$ in rational exponent form.

12.

Write $(3a)^{2/7}$ in radical form.

13.

Write $\sqrt[4]{10x^3}$ in rational exponent form.

14.

$$x^{1/3} \cdot x^{5/3}$$

15.

$$y^{7/4} \div y^{3/4}$$

16.

$$(m^{3/5})^{10/3}$$

17.

$$(2p^{1/4})(5p^{3/4})$$

18.

$$r^{1/2} \cdot r^{-3/2}$$

19.

$$(x^{4/3})^{-3/4}$$

20.

$$\text{Solve: } x^{5/3} = 32$$

21.

$$\text{Solve: } x^{2/3} = 16$$

22.

$$\text{Solve: } x^{3/2} = 27$$

23.

$$\text{Solve: } x^{-3/4} = 1/8$$

24.

$$\text{Solve: } x^{1/2} + 3 = 10$$

25.

$$\text{Solve: } \sqrt{x} = -5$$

Topic 1: Detailed Solutions

1.
Identify: Exponent $1/5$ — denominator 5 means fifth root.
Apply: $32^{1/5} = \sqrt[5]{32}$
Evaluate: $32 = 2^5$, so $\sqrt[5]{32} = 2$.
Verify: $2^5 = 32$ ✓
Answer: 2
2.
Identify: Exponent $2/3$ — index 3, power 2.
Apply: $27^{2/3} = (\sqrt[3]{27})^2$
Evaluate: $\sqrt[3]{27} = 3$, so $3^2 = 9$.
Verify: $9^{3/2} = (\sqrt{9})^3 = 27$ ✓
Answer: 9
3.
Identify: Exponent $3/4$ — index 4, power 3.
Apply: $16^{3/4} = (\sqrt[4]{16})^3$
Evaluate: $\sqrt[4]{16} = 2$, so $2^3 = 8$.
Verify: $8^{4/3} = (\sqrt[3]{8})^4 = 16$ ✓
Answer: 8
4.
Identify: Base is -8 ; exponent $2/3$ (odd denominator — OK).
Apply: $(-8)^{2/3} = (\sqrt[3]{-8})^2$
Evaluate: $\sqrt[3]{-8} = -2$, so $(-2)^2 = 4$.
Verify: $4^{3/2} = (\sqrt{4})^3 = 8$; check $(-8)^{2/3} = 4$ ✓
Answer: 4
5.
Identify: Negative exponent — take reciprocal; exponent $-1/2$.
Apply: $81^{-1/2} = 1 / 81^{1/2} = 1 / \sqrt{81}$
Evaluate: $\sqrt{81} = 9$, so the result is $1/9$.
Verify: $(1/9)^{-1} = 9 = \sqrt{81}$ ✓
Answer: $1/9$
6.
Identify: Exponent $3/2$ — index 2 (square root), power 3.
Apply: $100^{3/2} = (\sqrt{100})^3$
Evaluate: $\sqrt{100} = 10$, so $10^3 = 1000$.
Answer: 1000
7.
Identify: Exponent $2/3$ — index 3, power 2.
Apply: $64^{2/3} = (\sqrt[3]{64})^2$
Evaluate: $\sqrt[3]{64} = 4$, so $4^2 = 16$.
Verify: $16^{3/2} = (\sqrt{16})^3 = 64$ ✓
Answer: 16

8.

Identify: Base -125 , exponent $1/3$ (odd denominator — OK).

Apply: $(-125)^{1/3} = \sqrt[3]{-125}$

Evaluate: $(-5)^3 = -125$, so the cube root is -5 .

Verify: $(-5)^3 = -125$ ✓

Answer: -5

9.

Identify: Negative exponent — reciprocal first.

Apply: $9^{-3/2} = 1 / 9^{3/2} = 1 / (\sqrt{9})^3$

Evaluate: $\sqrt{9} = 3$, so $1 / 3^3 = 1/27$.

Verify: $(1/27)^{-2/3} = 27^{2/3} = 9$ ✓

Answer: $1/27$

10.

Identify: Rational exponent $4/5$.

Apply rule: Denominator $5 \rightarrow$ index; numerator $4 \rightarrow$ power.

Convert: $b^{4/5} = \sqrt[5]{(b^4)}$

Answer: $\sqrt[5]{(b^4)}$

11.

Identify: Index of radical = 6 ; exponent on radicand = 5 .

Apply rule: Index $\rightarrow$ denominator; exponent $\rightarrow$ numerator.

Convert: $\sqrt[6]{(c^5)} = c^{5/6}$

Answer: $c^{5/6}$

12.

Identify: Entire base $(3a)$ is raised to $2/7$.

Apply rule: The $(3a)$ is the radicand; exponent $2/7$ applies to all of it.

Convert: $(3a)^{2/7} = \sqrt[7]{((3a)^2)}$

Answer: $\sqrt[7]{((3a)^2)}$

13.

Identify: Index = 4 ; exponent on $x = 3$; coefficient 10 stays inside.

Apply rule: $\sqrt[4]{(10x^3)} = (10x^3)^{1/4}$

Answer: $(10x^3)^{1/4}$

14.

Identify: Same base x ; both exponents have denominator 3 .

Apply Product Rule: Add numerators: $1/3 + 5/3 = 6/3 = 2$.

Simplify: x^2

Answer: x^2

15.

Identify: Same base y ; both exponents have denominator 4 .

Apply Quotient Rule: Subtract exponents: $7/4 - 3/4 = 4/4 = 1$.

Simplify: $y^1 = y$

Answer: y

16.

Identify: Power of a power: $(m^{3/5})^{10/3}$.

Apply Power Rule: Multiply exponents: $(3/5) \times (10/3) = 30/15 = 2$.

Simplify: m^2

Answer: m^2

17.

Identify: Product of two powers of p with same denominator.

Combine coefficients: $2 \times 5 = 10$.

Apply Product Rule: Add exponents: $1/4 + 3/4 = 4/4 = 1$.

Simplify: $10p^1 = 10p$

Answer: $10p$

18.

Identify: Product of two powers of r with same denominator.

Apply Product Rule: Add exponents: $1/2 + (-3/2) = -2/2 = -1$.

Simplify: $r^{-1} = 1/r$ (negative exponent $\rightarrow$ reciprocal).

Answer: $1/r$

19.

Identify: Power of a power with a negative outer exponent.

Apply Power Rule: Multiply exponents: $(4/3) \times (-3/4) = -12/12 = -1$.

Simplify: $x^{-1} = 1/x$

Answer: $1/x$

20.

Identify: Equation $x^{5/3} = 32$; exponent 5/3, reciprocal 3/5.

Apply: Raise both sides to 3/5: $x = 32^{3/5}$.

Evaluate: $32^{3/5} = (\sqrt[5]{32})^3 = 2^3 = 8$.

Verify: $8^{5/3} = (\sqrt[3]{8})^5 = 2^5 = 32 \checkmark$

Answer: $x = 8$

21.

Identify: Equation $x^{2/3} = 16$; reciprocal of 2/3 is 3/2.

Apply: Raise both sides to 3/2: $x = 16^{3/2}$.

Evaluate: $16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$.

Verify: $64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16 \checkmark$

Answer: $x = 64$

22.

Identify: Equation $x^{3/2} = 27$; reciprocal of 3/2 is 2/3.

Apply: Raise both sides to 2/3: $x = 27^{2/3}$.

Evaluate: $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$.

Verify: $9^{3/2} = (\sqrt{9})^3 = 3^3 = 27 \checkmark$

Answer: $x = 9$

23.

Identify: Negative exponent equation: $x^{-3/4} = 1/8$.

Rewrite: $x^{-3/4} = 8^{-1}$, so $x^{3/4} = 8$ (take reciprocals of both sides).

Apply: Raise both sides to 4/3: $x = 8^{4/3}$.

Evaluate: $8^{4/3} = (\sqrt[3]{8})^4 = 2^4 = 16$.

Verify: $16^{-3/4} = 1/(16^{3/4}) = 1/8$ ✓

Answer: $x = 16$

24.

Identify: Equation with isolated radical plus a constant: $x^{1/2} + 3 = 10$.

Isolate: Subtract 3: $x^{1/2} = 7$.

Apply: Square both sides: $x = 7^2 = 49$.

Verify: $\sqrt{49} + 3 = 7 + 3 = 10$ ✓

Answer: $x = 49$

25.

Identify: Equation $\sqrt{x} = -5$.

Key fact: The principal square root $\sqrt{x}$ is always ≥ 0 for $x \geq 0$; it can never equal a negative number.

Conclusion: No real solution exists.

Verify: Squaring both sides would give $x = 25$, but $\sqrt{25} = 5 \neq -5$. Any apparent solution is extraneous.

Answer: No real solution

■ Watch Out! Common Mistakes in Topic 1

- 1. Sign errors with negative exponents.** A negative exponent means *reciprocal*, not a negative answer. $9^{-3/2} \neq -27$; the correct value is $1/27$.
- 2. Forgetting odd vs even index.** $(-8)^{2/3}$ is real ($= 4$), but $(-8)^{1/2}$ is not real — you cannot take an even root of a negative number in the reals.
- 3. Mixing up numerator and denominator.** $a^{m/n}$ means $\sqrt[n]{(a^m)}$ — the *denominator* is the *index*, the *numerator* is the *power*. Reversing them gives a completely different answer.
- 4. Skipping the check.** Equations with even-index roots can produce extraneous solutions. Always substitute your final answer back into the *original* equation.

Topic 2: Radical Expressions & Operations

Guided Notes

Simplifying nth Roots & Product/Quotient Rules

Product Rule for Radicals: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Rule for Radicals: $\sqrt[n]{a/b} = \sqrt[n]{a} \div \sqrt[n]{b}$

To simplify a radical, factor the radicand so that one factor is the largest perfect nth power. For example, $\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$. A radical is fully simplified when (1) the radicand has no perfect nth-power factor > 1 , (2) no fraction appears inside a radical, and (3) no radical appears in a denominator.

Example 1. Simplify: $\sqrt{50} + \sqrt{18}$

- **Identify.** Two radical terms; check whether they can be combined.
- **Simplify $\sqrt{50}$.** $50 = 25 \cdot 2$, so $\sqrt{50} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$.
- **Simplify $\sqrt{18}$.** $18 = 9 \cdot 2$, so $\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$.
- **Combine like radicals.** $5\sqrt{2} + 3\sqrt{2} = (5+3)\sqrt{2} = 8\sqrt{2}$.

✓ **Final Answer:** $8\sqrt{2}$

Example 2. Simplify: $\sqrt[3]{54}$

- **Identify.** Cube root; need the largest perfect-cube factor of 54.
- **Factor.** $54 = 27 \cdot 2$ and $27 = 3^3$.
- **Apply Product Rule.** $\sqrt[3]{54} = \sqrt[3]{(27 \cdot 2)} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$.

✓ **Final Answer:** $3\sqrt[3]{2}$

Multiplying Radical Expressions (FOIL)

When multiplying binomial radical expressions, use the FOIL (First, Outer, Inner, Last) method just as you do with standard binomials. After distributing, simplify each resulting radical and combine like radical terms if possible.

Example 3. Multiply: $(\sqrt{3} + 4)(\sqrt{3} - 5)$

- **Identify.** Two binomials with radicals — use FOIL.
- **First (F).** $\sqrt{3} \cdot \sqrt{3} = 3$
- **Outer (O).** $\sqrt{3} \cdot (-5) = -5\sqrt{3}$
- **Inner (I).** $4 \cdot \sqrt{3} = 4\sqrt{3}$
- **Last (L).** $4 \cdot (-5) = -20$
- **Combine.** $3 - 5\sqrt{3} + 4\sqrt{3} - 20$
- **Simplify.** $(3 - 20) + (-5\sqrt{3} + 4\sqrt{3}) = -17 - \sqrt{3}$.

✓ **Final Answer:** $-17 - \sqrt{3}$

Topic 4: Complex Numbers

Guided Notes

Defining the Imaginary Unit i

The imaginary unit i is defined as $i = \sqrt{-1}$, which means $i^2 = -1$.

A **complex number** is written in standard form as $a + bi$, where a is the **real part** and b is the **imaginary part**. If $b = 0$, the number is purely real. If $a = 0$ and $b \neq 0$, the number is purely imaginary. The set of real numbers is a subset of the complex numbers.

Powers of i

The Cycle of i

The powers of i repeat in a cycle of 4:

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

To simplify i^n for any positive integer n , divide n by 4. The remainder r tells you which power it reduces to: $i^n = i^r$. This works because $i^4 = 1$, so $i^{4k} = 1$ for any integer k .

Example 1. Simplify: i^{23}

→ **Identify.** Need to reduce i^{23} using the cycle of 4.

→ **Divide.** $23 \div 4 = 5$ remainder 3.

→ **Rewrite.** $i^{23} = i^{(4 \cdot 5 + 3)} = (i^4)^5 \cdot i^3$

→ **Apply $i^4 = 1$.** $1^5 \cdot i^3 = i^3$

→ **Use cycle.** $i^3 = -i$

✓ **Final Answer:** $-i$

Operations with Complex Numbers

Adding & Subtracting: Combine the real parts together and the imaginary parts together (treat i like a variable).

Multiplying: Use FOIL (or distributive property) just like binomials, then replace i^2 with -1 and combine like terms.

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