

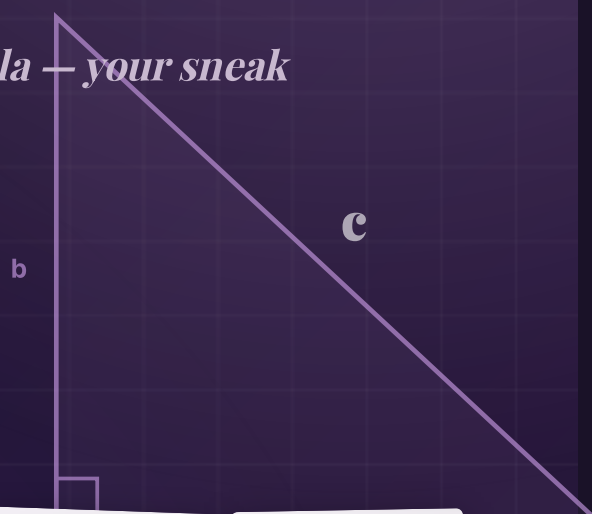
GRADE 8 • CCSS 8.G.B.6 / .7 / .8

Pythagorean Theorem

Complete Unit.

Proofs, applications & the distance formula — your sneak peek inside.

$$a^2 + b^2 = c^2 \quad \text{THEOREM}$$



A *glimpse* inside — 5 of 81 pages



Get the *full 81-page unit.*

3 proofs • 100+ worked problems • quizzes • test • guided notes

UNLOCK ALL →

#	Topic	CCSS	What Students Will Be Able To Do
3	Finding Unknown Sides	8.G.B.7	Solve for hypotenuse and legs; give exact radical and decimal forms.
4	Real-World Applications	8.G.B.7	Apply the theorem to 2D (ladders, screens, fields) and 3D (boxes, pyramids) contexts.
5	Distance Formula	8.G.B.8	Derive $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ from the theorem; find perimeters and verify right triangles.

How to Use This Resource

For teachers: Each topic opens with a SectionBar labelled by CCSS standard. Worked examples follow a consistent four-step protocol: (1) write the theorem symbolically; (2) substitute known values; (3) show every algebraic step; (4) give the exact radical form first, then the decimal approximation. Project these on a board and walk through them, then assign the corresponding drill or application set for independent practice. The two quizzes and the unit test are formatted to be printed double-sided and used as-is.

For students: Use the Guided Notes (after Topic 5) as a printable companion. Fill in the vocabulary, complete the partial proofs, and use the formula reference boxes as a quick lookup when solving problems. Every problem set has fully worked solutions immediately following — but try every problem on your own first, and only check the solution after you have made a genuine attempt.

For parents and tutors: The full solutions are written so that a non-expert can follow each step. If a student is stuck, read the worked solution aloud together, cover it, and have the student reproduce the steps from memory. This active recall is far more effective than passive reading.

Interactive Feature — Try-It Prompts

Throughout this unit you will see **Try-It** boxes that pause the exposition and ask you to attempt a small problem before reading the worked solution. These prompts are not optional — they are the single most important learning tool in the resource. Research in mathematics education consistently shows that students who attempt problems before seeing solutions retain the material far longer than students who simply read worked examples passively.

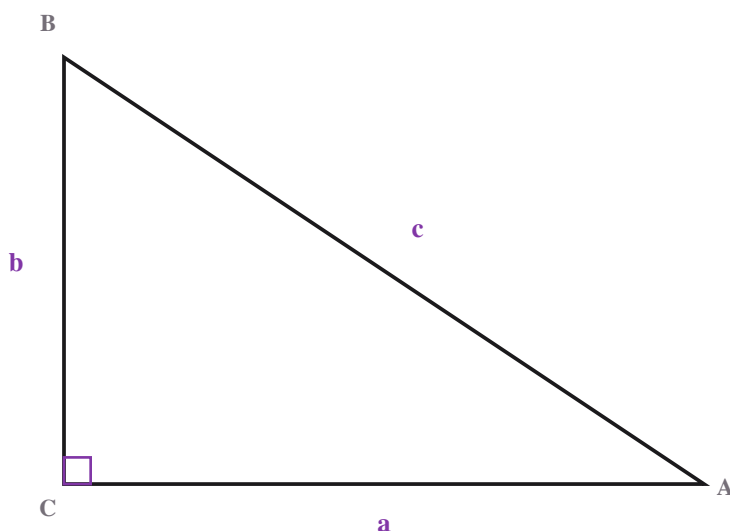


Figure 1.1 — A right triangle with legs a , b and hypotenuse c . The right angle is marked with a small square at vertex C .

The Converse — Working Backwards

The Pythagorean Theorem says: "If a triangle is a right triangle, then $a^2 + b^2 = c^2$." The **converse** reverses the implication: "If a triangle has side lengths satisfying $a^2 + b^2 = c^2$, then it is a right triangle (with the right angle opposite the side of length c)." The converse is also true, and it gives us a powerful tool — we can verify that an angle is a right angle just by checking the side lengths, without ever using a protractor.

CONVERSE (8.G.B.6)

Converse of the Pythagorean Theorem: If a triangle has sides a , b , c and $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Worked Example 1.1 — Identify the Hypotenuse

Problem. A right triangle has sides of length 7, 24, and 25. Which side is the hypotenuse? Verify that the side lengths satisfy the Pythagorean Theorem.

Solution. The hypotenuse is always the *longest* side, so $c = 25$. The legs are $a = 7$ and $b = 24$.

Step 1. Write the theorem: $a^2 + b^2 = c^2$

Step 2. Substitute: $7^2 + 24^2 = 25^2$

Step 3. Compute each square: $49 + 576 = 625$

Step 4. Add the left side: $49 + 576 = 625$ ✓

Conclusion. The triangle is a right triangle with hypotenuse 25. The triple (7, 24, 25) is a Pythagorean triple.

Topic 2 · Proof 1: Square-in-a-Square Dissection

CCSS 8.G.B.6

Our first proof is the **Bhaskara dissection**, sometimes called the "square-in-a-square" proof. The idea is breathtakingly simple: arrange four identical right triangles inside a larger square so that they form a smaller tilted square in the middle. We compute the area of the large square in two different ways — once using its side length, and once by adding the areas of the four triangles plus the inner square. Setting the two computations equal gives us the theorem.

Setup and Diagram

Take four identical right triangles, each with legs a and b and hypotenuse c . Arrange them corner-to-corner inside a square whose side length is $(a + b)$, as shown below. The four hypotenuses form the boundary of an inner tilted square whose side length is c .

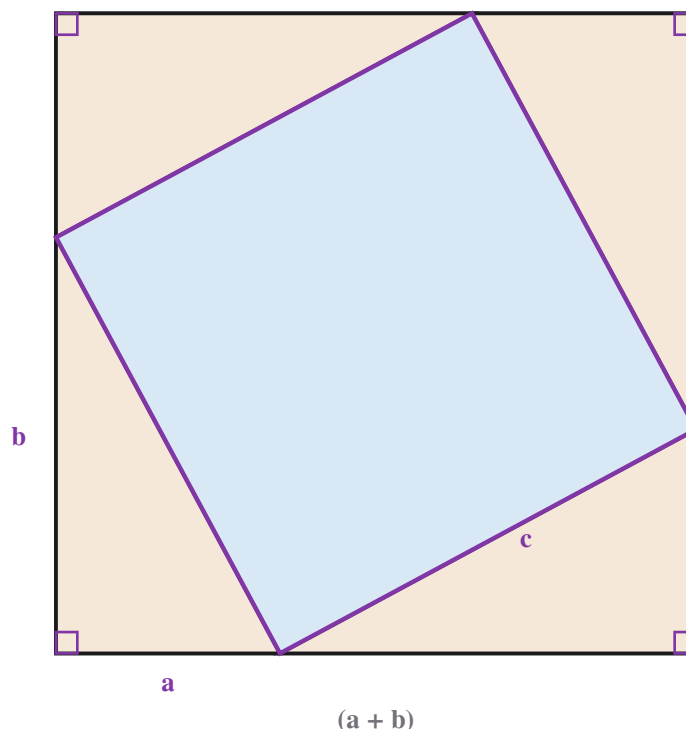


Figure 2.1 — Four congruent right triangles arranged inside a square of side $(a + b)$. The inner tilted square has side c .

Algebraic Proof

Step 1 — Compute the area of the outer square two ways.

The outer square has side length $(a + b)$, so its area is:

$$\text{Area(outer)} = (a + b)^2$$

Topic 2 · Proof 3: Garfield's Trapezoid

CCSS 8.G.B.6

Our third proof was discovered in 1876 by James A. Garfield — then a U.S. Congressman, later the 20th President of the United States. Garfield's proof uses a **trapezoid** divided into three right triangles. Like the square-in-square proof, it computes the same area two different ways and equates the results. The novelty is that one of the three triangles is an **isosceles right triangle** whose legs are the hypotenuses of the two original right triangles — and this isosceles triangle's area formula is what produces the c^2 term.

Setup and Diagram

Take two congruent right triangles, each with legs a and b and hypotenuse c . Place them as shown in Figure 2.3 so that they meet at a single point along the bottom of a trapezoid. The trapezoid has parallel vertical sides of lengths a and b , and the bottom side has length $(a + b)$. A third triangle — an isosceles right triangle with legs of length c — fills the remaining space at the top of the trapezoid.

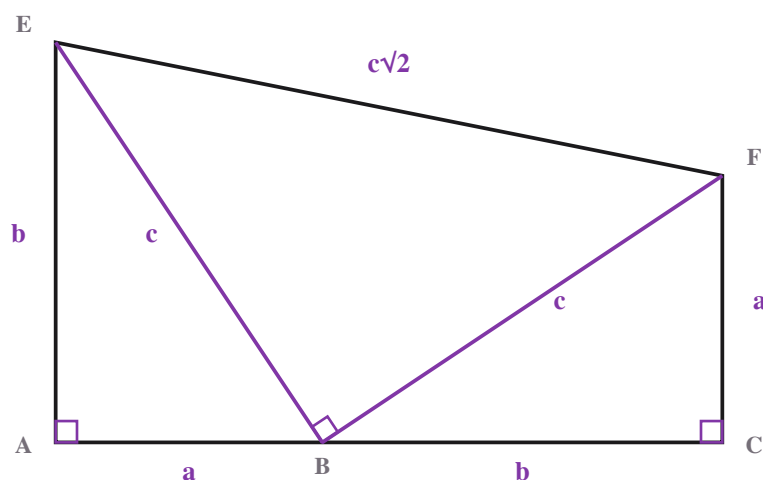


Figure 2.3 — Garfield's trapezoid AEFC. Triangle ABE (right angle at A) and triangle BCF (right angle at C) are congruent right triangles with legs a , b . Triangle BEF is an isosceles right triangle with legs c , c .

Algebraic Proof

Step 1 — Compute the area of the trapezoid by its formula.

The trapezoid AEFC has parallel sides $AE = b$ and $CF = a$, and the perpendicular distance between them (the "height" of the trapezoid, measured horizontally) is $AC = a + b$. The area of a trapezoid is $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$:

$$\text{Area}(\text{trapezoid}) = \frac{1}{2} \cdot (a + b) \cdot (a + b) = \frac{1}{2} (a + b)^2$$

Topic 3: Finding Unknown Sides

CCSS 8.G.B.7

Once we accept that $a^2 + b^2 = c^2$ holds in every right triangle, we can use the equation as a computational tool. Given the lengths of any two sides of a right triangle, the third side is determined. There are two cases: **finding the hypotenuse** (when the two legs are known) and **finding a leg** (when one leg and the hypotenuse are known). The protocol is the same in both cases — write the theorem symbolically, substitute, solve — but the algebra differs slightly because finding a leg requires subtracting before taking a square root.

The Four-Step Solution Protocol

Step 1. Write the theorem: $a^2 + b^2 = c^2$

Step 2. Substitute the known values.

Step 3. Solve algebraically for the unknown side.

Step 4. Give the **exact radical form** first, then the **decimal approximation**.

Worked Example 3.1 — Finding the Hypotenuse (Perfect Square)

Problem. A right triangle has legs of length 6 and 8. Find the hypotenuse.

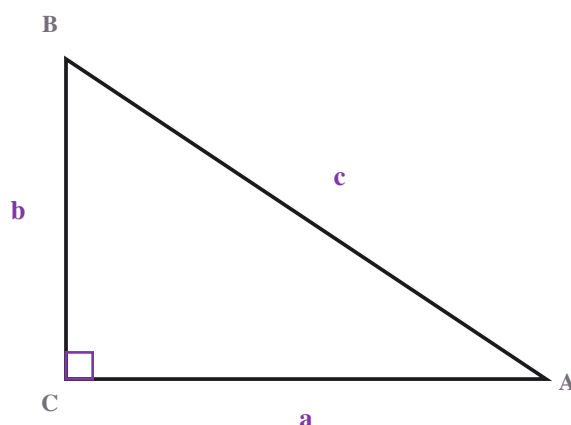


Figure 3.1 — Right triangle with legs 6 and 8; find the hypotenuse c .

Step 1. $a^2 + b^2 = c^2$

Step 2. $6^2 + 8^2 = c^2$

Step 3. $36 + 64 = c^2$

Step 4. $100 = c^2$

Step 5. $c = \sqrt{100} = 10$

Exact form: $c = 10$. Decimal: $c = 10.0$

Note: (6, 8, 10) is a multiple of the (3, 4, 5) Pythagorean triple — specifically, $2 \times (3, 4, 5)$.

Worked Example 3.2 — Finding a Leg (Perfect Square)

Base Triple	Verification	Common Multiples
(20, 21, 29)	$400 + 441 = 841$	(40, 42, 58); (60, 63, 87); (80, 84, 116)

Try-It 3.A — Recognize the Triple

Without computing any squares, identify the missing side in each right triangle: (a) legs 9 and 12, find c . (b) leg 15 and hypotenuse 17, find the other leg. (c) legs 24 and 32, find c . (d) legs 14 and 48, find c .

Try-It 3.A Answers. (a) $c = 15$ (triple 3-4-5 $\times 3$). (b) Other leg = 8 (triple 8-15-17). (c) $c = 40$ (triple 3-4-5 $\times 8$). (d) $c = 50$ (triple 7-24-25 $\times 2$).

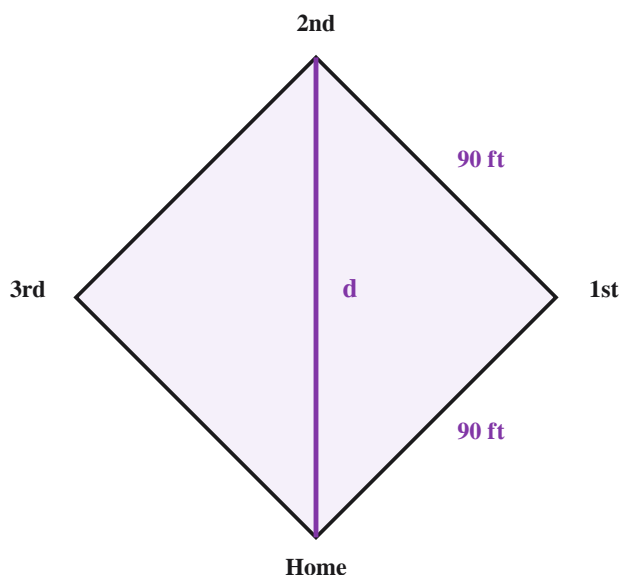


Figure 4.3 — The baseball diamond is a square. The straight-line distance from home to 2nd base is the diagonal of the square.

Solution. The diagonal of the square baseball diamond divides it into two right triangles, each with legs of 90 feet and hypotenuse equal to the home-to-second distance.

Step 1. $a^2 + b^2 = c^2$

Step 2. $90^2 + 90^2 = c^2$

Step 3. $8100 + 8100 = c^2$

Step 4. $16200 = c^2$

Step 5. $c = \sqrt{16200} = \sqrt{(8100 \cdot 2)} = 90\sqrt{2}$

Exact: $c = 90\sqrt{2}$ ft. Decimal: $c \approx 127.28$ ft

Answer: The straight-line distance from home to second base is $90\sqrt{2} \approx 127.28$ feet. Note: this is the diagonal of a square with side s ; it always equals $s\sqrt{2}$.

Worked Example 4.4 — TV Screen Size

Problem. A 50-inch television has an aspect ratio of 16:9, meaning the width-to-height ratio is 16 to 9. Find the actual width and height of the screen. (The "50-inch" measurement refers to the diagonal of the screen.)

Topic 4 · Real-World 3D Applications

CCSS 8.G.B.7

The Pythagorean Theorem extends naturally into three dimensions. The key insight is that a 3D problem almost always reduces to a sequence of 2D right-triangle problems. We apply the theorem twice (or sometimes three times), each time to a different right triangle whose sides are lengths we either know or have just computed. The two most important 3D formulas derived this way are the **space diagonal of a rectangular box**, $d = \sqrt{a^2 + b^2 + c^2}$, and the **height of a pyramid from its slant height**, $h = \sqrt{\ell^2 - (s/2)^2}$, where ℓ is the slant height and s is the side length of the square base.

Worked Example 4.5 — Space Diagonal of a Box

Problem. A rectangular box measures 3 feet by 4 feet by 12 feet. Find the length of the space diagonal — the straight line from one corner of the box to the opposite corner.

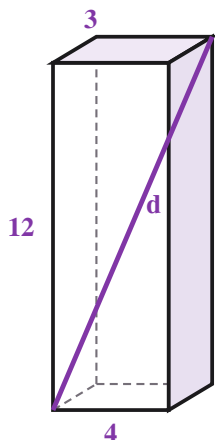


Figure 4.5 — A rectangular box with dimensions $3 \times 4 \times 12$. The space diagonal runs from the front-bottom-left corner to the back-top-right corner.

Solution. The space diagonal is found by applying the Pythagorean Theorem twice. First, find the diagonal of the base (a 2D right triangle with legs 3 and 4). Then use that diagonal as one leg of a second right triangle whose other leg is the height of the box (12) and whose hypotenuse is the space diagonal.

Step 1 — Diagonal of the base.

$$\begin{aligned} 3^2 + 4^2 &= d_{\text{base}}^2 \\ 9 + 16 &= d_{\text{base}}^2 \\ d_{\text{base}}^2 &= 25 \rightarrow d_{\text{base}} = 5 \text{ ft} \end{aligned}$$

Step 2 — Space diagonal. Now we have a right triangle with one leg = $d_{\text{base}} = 5$ ft, the other leg = height of box = 12 ft, and the hypotenuse = space diagonal D .

$$\begin{aligned} 5^2 + 12^2 &= D^2 \\ 25 + 144 &= D^2 \\ 169 &= D^2 \end{aligned}$$

Topic 5: The Distance Formula

CCSS 8.G.B.8

The Pythagorean Theorem has a famous alter ego: the **distance formula**. Whenever two points live in a coordinate plane, we can construct a right triangle whose legs are parallel to the x - and y -axes and whose hypotenuse is the segment connecting the two points. Applying the theorem to that right triangle gives us a formula for the distance between any two coordinate points. The two formulas are not merely related — they are the same statement in different clothing.

Derivation of the Distance Formula

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the coordinate plane. To find the distance between them, we construct a right triangle with P and Q as two of the vertices. The right angle is at the point $R(x_2, y_1)$ — directly below (or above) Q and directly to the right (or left) of P . The horizontal leg PR has length $|x_2 - x_1|$, the vertical leg RQ has length $|y_2 - y_1|$, and the hypotenuse PQ is the distance d we want to find.

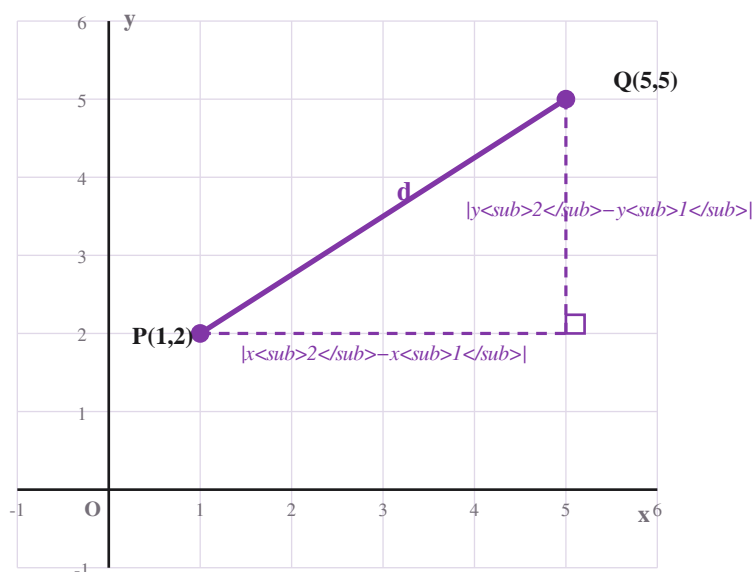


Figure 5.1 — The distance formula is the Pythagorean Theorem applied to the right triangle PQR .

Step 1 — Apply the Pythagorean Theorem to $\triangle PQR$.

$$(PR)^2 + (RQ)^2 = (PQ)^2$$

Step 2 — Substitute the leg lengths.

$$|x_2 - x_1|^2 + |y_2 - y_1|^2 = d^2$$

Step 3 — Use the fact that $|w|^2 = w^2$ for any real number w .

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

Student Visual Guided Notes

CCSS 8.G.B.6 / 8.G.B.7 / 8.G.B.8

Print the following pages and complete them by hand as you work through the unit. The guided notes are designed to be a permanent reference — once filled in, they become a one-page summary of the entire unit that you can use to study for quizzes and the unit test. Three blank proof diagrams are provided so you can practice reconstructing the proofs from memory.

Part A • Vocabulary Fill-Ins

Complete each definition with the correct term.

Statement	Answer
The longest side of a right triangle, opposite the right angle, is called the _____.	_____
The two shorter sides of a right triangle that form the right angle are called the _____.	_____
The Pythagorean Theorem states that $a^2 + b^2 = \underline{\hspace{1cm}}$.	_____
The converse says: if $a^2 + b^2 = c^2$, then the triangle is a _____ triangle.	_____
A set of three positive integers (a, b, c) satisfying $a^2 + b^2 = c^2$ is called a _____.	_____
The distance formula is $d = \sqrt{[(x_2 - x_1)^2 + (\underline{\hspace{1cm}} - \underline{\hspace{1cm}})^2]}$.	_____

Part B • Formula Reference Boxes

Copy each formula into the box below. These are the four formulas you must memorize.

Pythagorean Theorem 	Converse of the Pythagorean Theorem
Distance Formula 	Simplified Radical Form

Drill Set A: 40 Unknown-Side Calculations**CCSS 8.G.B.7**

This drill contains 40 problems in two groups: problems 1–20 ask you to find the hypotenuse given the two legs; problems 21–40 ask you to find a leg given the hypotenuse and the other leg. For each problem, write the theorem symbolically, substitute, solve algebraically, and give both the exact radical form and the decimal approximation rounded to two decimal places. Many problems will resolve to Pythagorean triples; some will give irrational answers that must be left in simplified radical form.

Part A • Find the Hypotenuse (Problems 1–20)

Given the two legs a and b , find the hypotenuse c .

#1. Legs $a = 3$, $b = 4$. Find c .

#11. Legs $a = 10$, $b = 24$. Find c .

#2. Legs $a = 5$, $b = 12$. Find c .

#12. Legs $a = 12$, $b = 16$. Find c .

#3. Legs $a = 8$, $b = 15$. Find c .

#13. Legs $a = 15$, $b = 20$. Find c .

#4. Legs $a = 7$, $b = 24$. Find c .

#14. Legs $a = 9$, $b = 40$. Find c .

#5. Legs $a = 9$, $b = 12$. Find c .

#15. Legs $a = 11$, $b = 60$. Find c .

#6. Legs $a = 6$, $b = 8$. Find c .

#16. Legs $a = 5$, $b = 2$. Find c .

#7. Legs $a = 1$, $b = 1$. Find c .

#17. Legs $a = 8$, $b = 8$. Find c .

#8. Legs $a = 5$, $b = 5$. Find c .

#18. Legs $a = 6$, $b = 11$. Find c .

#9. Legs $a = 2$, $b = 3$. Find c .

#19. Legs $a = 14$, $b = 48$. Find c .

#10. Legs $a = 4$, $b = 7$. Find c .

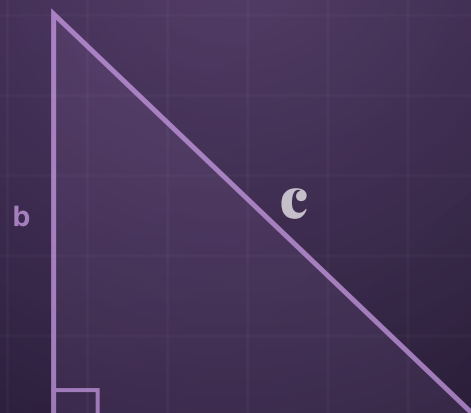
#20. Legs $a = 3$, $b = 6$. Find c .

Comprehensive Unit Test**CCSS 8.G.B.6 / 8.G.B.7 / 8.G.B.8****Total points: 100 · Time: 70 minutes · Show all work for full credit.****Part A · Multiple Choice (15 pts, 3 pts each)**

1. In a right triangle with legs 5 and 12, the hypotenuse is: (a) 7 (b) 13 (c) 17 (d) $\sqrt{119}$
2. Which of the following is a Pythagorean triple? (a) (2, 3, 4) (b) (4, 5, 6) (c) (8, 15, 17) (d) (5, 7, 9)
3. The space diagonal of a $2 \times 3 \times 6$ box is: (a) 7 (b) 11 (c) $\sqrt{55}$ (d) 8
4. The distance between (0, 0) and (3, 4) is: (a) 7 (b) 5 (c) $\sqrt{7}$ (d) 25
5. In the square-in-square proof, the side of the inner square is: (a) $a+b$ (b) a (c) b (d) c

You've seen the *sneak peek.*

Now unlock all 81 pages of proofs, applications, drills, quizzes & the full unit test.

**81**

PAGES

5

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3

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Inside the *full unit*

EVERYTHING BELOW IS INCLUDED IN YOUR DOWNLOAD

**Topic 1 · Statement & Vocabulary****Topic 2 · Three Full Proofs (Bhaskara, Similar, Garfield)****Topic 3 · Finding Unknown Sides (4-step protocol)****Topic 4 · 35 Real-World Applications (2D + 3D)****Topic 5 · Distance Formula Derivation****Student Visual Guided Notes (printable)****40-Problem Unknown-Side Drill + Solutions****20-Problem 2D Applications + Solutions****15-Problem 3D Applications + Diagrams****25-Problem Coordinate Distance Drill****Cumulative 30-Problem Review +****2 Quizzes + Comprehensive Unit**

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