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Matrix Fundamentals & Operations

What You Will Learn in Topic 1

By the end of this topic, you will be able to:

- Name the dimensions of any matrix and identify any entry using subscript notation (e.g. a_{23})
- Add and subtract matrices by combining matching entries
- Multiply a matrix by a scalar (single number)
- Recognise and use the zero matrix and identity matrix

1.1 What Is a Matrix?

• Definition: Matrix

A **matrix** is a rectangular array of numbers arranged in rows and columns. We write a matrix inside square brackets.

A matrix with m rows and n columns is called an $m \times n$ **matrix** (read "m by n").

Consider a store that sells three product types in two locations. The inventory data can be organized as a matrix:

$$I = \begin{bmatrix} 120 & 85 & 64 \\ 95 & 110 & 73 \end{bmatrix}$$

Product A Product B Product C

← Store 1
← Store 2

1.2 Dimensions & Entry Notation

We denote a matrix by a capital letter (e.g. A) and write its dimensions as $\mathbf{A}_{m \times n}$, where m = number of rows and n = number of columns.

• Entry Notation

The entry in row i , column j of matrix A is written $\mathbf{a}_{ij}$ (read "a sub i-j").

Row index i counts from top (1, 2, 3, ...). Column index j counts from left (1, 2, 3, ...).

Guided Note: A matrix with 3 rows and 4 columns is a 3×4 matrix. The entry in the 2nd row, 3rd column of matrix B is written b_{23} .

• **Example 1: Identifying Dimensions & Entries**

Given the matrix:

$$\mathbf{A} = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 0 & -5 \end{bmatrix}$$

Step 1 — Dimensions: A has 2 rows and 3 columns, so A is a **2 × 3 matrix**.

Step 2 — Identify entries by position:

$$a_{11} = 3, a_{12} = -1, a_{13} = 4$$

$$a_{21} = 2, a_{22} = 0, a_{23} = -5$$

1.3 Matrix Addition & Subtraction

• **Addition & Subtraction Rule**

Two matrices can be added (or subtracted) **only if they have the same dimensions**.

To add (subtract), combine corresponding entries:

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \text{ means } c_{ij} = a_{ij} + b_{ij}$$

$$\mathbf{C} = \mathbf{A} - \mathbf{B} \text{ means } c_{ij} = a_{ij} - b_{ij}$$

• Example 2: Matrix Addition

Find $A + B$, where:

$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$$

and

$$\mathbf{B} = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$$

Step 1 — Check dimensions: Both A and B are 2×2 . [OK] Same dimensions.

Step 2 — Add each corresponding pair:

$$c_{11} = 1 + 5 = 6 \quad c_{12} = 3 + (-1) = 2$$

$$c_{21} = -2 + 3 = 1 \quad c_{22} = 4 + 2 = 6$$

Step 3 — Write the result:

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 6 & 2 \\ 1 & 6 \end{bmatrix}$$

• Example 3: Matrix Subtraction

Find $C - D$, where:

$$\mathbf{C} = \begin{bmatrix} 7 & 2 \\ -1 & 3 \end{bmatrix}$$

and

$$\mathbf{D} = \begin{bmatrix} 3 & 5 \\ 2 & -4 \end{bmatrix}$$

Step 1 — Check dimensions: Both are 2×2 . [OK]

Step 2 — Subtract corresponding entries:

$$c_{11} - d_{11} = 7 - 3 = 4 \quad c_{12} - d_{12} = 2 - 5 = -3$$

$$c_{21} - d_{21} = -1 - 2 = -3 \quad c_{22} - d_{22} = 3 - (-4) = 7$$

Step 3 — Write the result:

$$\mathbf{C} - \mathbf{D} = \begin{bmatrix} 4 & -3 \\ -3 & 7 \end{bmatrix}$$

!! Dimension Mismatch!

If two matrices have different dimensions, their sum or difference is **undefined**. For example, a 2×3 matrix cannot be added to a 2×2 matrix.

1.4 Scalar Multiplication

- **Scalar Multiplication**

A **scalar** is a single number (not a matrix). To multiply a matrix by a scalar k , multiply **every entry** by k :

$k\mathbf{A}$ means $(k\mathbf{A})_{ij} = k \cdot a_{ij}$

- **Example 4: Scalar Multiplication**

Find $3\mathbf{A}$, where:

$$\mathbf{A} = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$$

Step 1 — Multiply each entry by 3:

$$3 \cdot 2 = 6 \quad 3 \cdot (-3) = -9$$

$$3 \cdot 1 = 3 \quad 3 \cdot 4 = 12$$

Step 2 — Write the result:

$$3\mathbf{A} = \begin{bmatrix} 6 & -9 \\ 3 & 12 \end{bmatrix}$$

• **Example 5: Combined Operations: $2A - 3B$**

Find $2A - 3B$, where:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$$

Step 1 — Compute $2A$:

$$2A = \begin{bmatrix} 2 & 4 \\ 6 & 0 \end{bmatrix}$$

Step 2 — Compute $3B$:

$$3B = \begin{bmatrix} -3 & 12 \\ 6 & 3 \end{bmatrix}$$

Step 3 — Subtract: $2A - 3B$:

$$2 - (-3) = 5 \quad 4 - 12 = -8$$

$$6 - 6 = 0 \quad 0 - 3 = -3$$

$$2A - 3B = \begin{bmatrix} 5 & -8 \\ 0 & -3 \end{bmatrix}$$

Verification: Check entry (1,1): $2(1) - 3(-1) = 2 + 3 = 5$ [OK]

1.5 Zero & Identity Matrices

• **Zero Matrix (O)**

The **zero matrix** $O_{m \times n}$ has every entry equal to 0. It is the additive identity:

$$A + O = O + A = A \text{ (for any matrix } A \text{ of the same dimensions)}$$

• **Identity Matrix (I_n)**

The **identity matrix** I_n is an $n \times n$ square matrix with 1s on the main diagonal and 0s everywhere else.

It is the multiplicative identity: $A \cdot I = I \cdot A = A$ (for any $n \times n$ matrix A).

The 2×2 and 3×3 identity matrices:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• Example 6: Properties of Zero & Identity Matrices

Given:

$$A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$$

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Part A: Find $A + O$

$$4+0=4, -2+0=-2, 3+0=3, 1+0=1$$

$$A + O = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$$

Result is A itself, confirming $A + O = A$. [OK]

Part B: Find $A \cdot I$ (Preview of Topic 2)

$$\text{Row 1} \cdot \text{Col 1: } (4)(1) + (-2)(0) = 4$$

$$\text{Row 1} \cdot \text{Col 2: } (4)(0) + (-2)(1) = -2$$

$$\text{Row 2} \cdot \text{Col 1: } (3)(1) + (1)(0) = 3$$

$$\text{Row 2} \cdot \text{Col 2: } (3)(0) + (1)(1) = 1$$

$$A \cdot I = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$$

Result is A itself, confirming $A \cdot I = A$. [OK]

Topic 1 Guided Practice

1. State the dimensions of a matrix with 4 rows and 2 columns.
2. If M is a 3×3 matrix, write the notation for the entry in row 1, column 3.
3. Find the sum:

$$\begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} -4 & 1 \\ 6 & -2 \end{bmatrix}$$

4. Find $4A$, where $A =$

$$\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix}$$

5. Find $2X - Y$, where:

$$X = \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$$

• Key Takeaways from Topic 1

- Matrices must have the **same dimensions** to be added or subtracted.
- Scalar multiplication distributes across the matrix: $k(A + B) = kA + kB$.
- The zero matrix O acts like the number 0 for addition.
- The identity matrix I acts like the number 1 for multiplication.

Topic 1 Quick-Reference Summary

Operation	Requirement	How To Apply	Example
Addition $A+B$	Same $m \times n$ dimensions	Add matching entries: $c_{ij} = a_{ij} + b_{ij}$	$[1,2]+[3,4]=[4,6]$
Subtraction $A-B$	Same $m \times n$ dimensions	Subtract matching entries: $c_{ij} = a_{ij} - b_{ij}$	$[5,3]-[2,1]=[3,2]$
Scalar kA	Any matrix	Multiply EVERY entry by k	$3*[2,-1]=[6,-3]$
Zero Matrix O	Same size as A	$A + O = A$ (additive identity)	Any $A + O = A$
Identity I	Square matrices only	$A*I = I*A = A$ (multiplicative identity)	Any square A

Topic 4 Guided Practice

1. Write the system as a matrix equation $AX = B$:

$$4x - y = 5$$

$$2x + 3y = 7$$

2. Solve the system using Cramer's Rule:

$$x + 5y = 3$$

$$2x - y = 4$$

3. Solve the system using Gaussian Elimination (RREF):

$$x + 2y + z = 3$$

$$2x - y + z = 0$$

$$3x + y + 2z = 3$$

• Key Takeaways from Topic 4

- $AX = B$ compresses an entire system into three matrices: Coefficients, Variables, Constants.
- If A is invertible, $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ provides the exact unique solution.
- **Cramer's Rule** is a determinant shortcut: $x_i = \det(A_i) / \det(A)$.
- **Gaussian Elimination** is the most robust method; it works even when A is not invertible.
- Always look at the RREF to identify if a system is Independent, Inconsistent, or Dependent.

Key Formulas — Topic 4

Matrix Equation: $AX = B$

Inverse Method: $X = A^{-1}B$ (multiply A^{-1} on the LEFT)

Cramer's Rule: $x_i = \det(A_i) / \det(A)$ where A_i replaces column i with B

Topic 4 Quick-Reference: Choosing a Solution Method

Method	Best Used When	Key Steps	Singular A?
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Inverse: $X = A^{-1}B$	Small system; A is invertible	1. Find A^{-1} 2. Multiply: $X = A^{-1}B$	No (need $\det \neq 0$)
Cramer's Rule	2x2 or 3x3; quick scalar answer	1. Find $\det(A)$ 2. Replace col i of A with B to get A_i 3. $x_i = \det(A_i) / \det(A)$	No (need $\det \neq 0$)
Gaussian Elimination / RREF	Any system, any size	1. Form $[A B]$ 2. Row-reduce to RREF 3. Read solution or identify type	Yes — handles all cases

!! !! Common Mistakes — Topic 4

Mistake 1 — Multiplying A^{-1} on the wrong side: From $AX = B$, the correct move is $A^{-1}(AX) = A^{-1}B$, giving $X = A^{-1}B$. Writing $XA^{-1} = BA^{-1}$ is incorrect because matrix multiplication is not commutative.

Mistake 2 — Replacing the wrong column in Cramer's Rule: To find x_1 , replace COLUMN 1 of A with vector B. To find x_2 , replace COLUMN 2. Replacing the wrong column gives a completely wrong answer.

Mistake 3 — Row operation applied to wrong row in RREF: Write each step clearly: " $R_2 + 3R_1 \rightarrow R_2$ " changes ONLY R_2 . Accidentally changing R_1 as well corrupts all subsequent steps.

Mistake 4 — Misreading RREF for system type: A row $[0 \ 0 \ 0 \ | \ 0]$ means the system is DEPENDENT (free variable, infinitely many solutions). A row $[0 \ 0 \ 0 \ | \ c]$ with $c \neq 0$ means INCONSISTENT (no solution). These two look similar — check the last column carefully.

gallons leave. Write the system of equations.

2. Product X requires 2 units of Wood and 1 unit of Metal. Product Y requires 3 units of Wood and 4 units of Metal. Wood costs \$5/unit, Metal costs \$8/unit. Use matrix multiplication to find the total cost of each product.

• Key Takeaways from Topic 5

- Matrices elegantly condense complex real-world systems (like traffic or economics) into solvable equations $AX = B$.
- **Inventory/Cost Analysis** uses matrix multiplication to link requirements to prices.
- **Leontief Models** use inverse matrices ($X = (I - A)^{-1}D$) to find total production for an economy.
- **Geometric Transformations** use matrix multiplication to alter coordinates, reading right-to-left.

Key Formulas — Topic 5

$$\text{Leontief Model: } X = AX + D \rightarrow X = (I - A)^{-1}D$$

Transformation Composition: "Apply T_1 first, then T_2 " = $T_2 \cdot T_1$ (T_1 on the right)

!! !! Common Mistakes — Topic 5

Mistake 1 — Transformation order (right-to-left rule): "Apply rotation R first, then reflection F " is written as the product $F \cdot R$ — the FIRST transformation applied goes on the RIGHT. Writing $R \cdot F$ gives the opposite composition and a completely different result.

Mistake 2 — Leontief: using A instead of $(I - A)$: The total output formula is $X = (I - A)^{-1}D$. You must first compute $I - A$, then find its inverse. Using A^{-1} directly is incorrect.

Mistake 3 — Network flow: wrong direction for equations: Conservation of flow requires Flow IN = Flow OUT at every node. Carefully identify which flows enter and which exit each node before writing the equation.

Mistake 4 — Cost matrix: wrong multiplication order: If P is (products $\times$ materials) and C is (materials $\times$ stores), then PC gives cost per product per store. Computing CP gives a dimensionally different result with no practical meaning.

Mistake 5 — Leontief: not solving $(I - A)D$ instead of $(I - A)^{-1}D$: The formula requires the INVERSE of $(I - A)$ multiplied by D , not just $(I - A)$ multiplied by D . Always find the inverse of $(I - A)$ first.

Practice & Assessment

Matrix Operations Drill Set

Complete the following matrix operations. Show all intermediate steps.

Problem 1: Find $A + B$.

$$\mathbf{A} = \begin{bmatrix} 4 & -1 \\ 2 & 5 \end{bmatrix} \quad \text{and}$$

$$\mathbf{B} = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$$

Solution 1:

Step 1: Check dimensions. Both are 2×2 . [OK]

Step 2: Add corresponding entries.

$$c_{11} = 4 + 3 = 7, \quad c_{12} = -1 + 2 = 1$$

$$c_{21} = 2 + (-1) = 1, \quad c_{22} = 5 + 4 = 9$$

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 7 & 1 \\ 1 & 9 \end{bmatrix}$$

Problem 2: Find $C - D$.

$$\mathbf{C} = \begin{bmatrix} -2 & 5 \\ 0 & 3 \end{bmatrix} \quad \text{and}$$

$$\mathbf{D} = \begin{bmatrix} -4 & 2 \\ 6 & -1 \end{bmatrix}$$

Solution 2:

Step 1: Check dimensions. Both are 2×2 . [OK]

Step 2: Subtract corresponding entries.

$$c_{11} = -2 - (-4) = 2, \quad c_{12} = 5 - 2 = 3$$

$$c_{21} = 0 - 6 = -6, \quad c_{22} = 3 - (-1) = 4$$

$$\mathbf{C} - \mathbf{D} = \begin{bmatrix} 2 & 3 \\ -6 & 4 \end{bmatrix}$$

Problem 3: Find $E + F$.

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 4 & 1 \end{bmatrix} \quad \text{and}$$

$$\mathbf{F} = \begin{bmatrix} -1 & 2 & 3 \\ 5 & -4 & 0 \end{bmatrix}$$

Solution 3:

Step 1: Check dimensions. Both are 2×3 . [OK]

Step 2: Add corresponding entries.

Row 1: $1 + (-1) = 0$, $0 + 2 = 2$, $-3 + 3 = 0$

Row 2: $2 + 5 = 7$, $4 + (-4) = 0$, $1 + 0 = 1$

$$\mathbf{E} + \mathbf{F} = \begin{bmatrix} 0 & 2 & 0 \\ 7 & 0 & 1 \end{bmatrix}$$

Problem 4: Find $\mathbf{G} - \mathbf{H}$.

$$\mathbf{G} = \begin{bmatrix} 8 & 2 \\ -3 & 1 \\ 0 & 5 \end{bmatrix} \quad \text{and}$$

$$\mathbf{H} = \begin{bmatrix} 5 & -2 \\ 3 & 1 \\ -4 & 2 \end{bmatrix}$$

Solution 4:

Step 1: Check dimensions. Both are 3×2 . [OK]

Step 2: Subtract corresponding entries.

Row 1: $8 - 5 = 3$, $2 - (-2) = 4$

Row 2: $-3 - 3 = -6$, $1 - 1 = 0$

Row 3: $0 - (-4) = 4$, $5 - 2 = 3$

$$\mathbf{G} - \mathbf{H} = \begin{bmatrix} 3 & 4 \\ -6 & 0 \\ 4 & 3 \end{bmatrix}$$

Problem 5: Find $3\mathbf{A}$.

$$\mathbf{A} = \begin{bmatrix} 2 & -4 \\ 0 & 5 \end{bmatrix}$$

Solution 5:

Step 1: Multiply every entry by 3.

$$3(2)=6, 3(-4)=-12, 3(0)=0, 3(5)=15$$

$$3\mathbf{A} = \begin{bmatrix} 6 & -12 \\ 0 & 15 \end{bmatrix}$$

Problem 6: Find $2\mathbf{X} - \mathbf{Y}$.

$$\mathbf{X} = \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix} \quad \text{and}$$

$$\mathbf{Y} = \begin{bmatrix} -2 & 1 \\ 0 & 5 \end{bmatrix}$$

Solution 6:

Step 1: Compute $2\mathbf{X}$.

$$2\mathbf{X} = [2, -6 ; 8, 4]$$

Step 2: Compute $2\mathbf{X} - \mathbf{Y}$.

$$c_{11} = 2 - (-2) = 4, c_{12} = -6 - 1 = -7$$

$$c_{21} = 8 - 0 = 8, c_{22} = 4 - 5 = -1$$

$$2\mathbf{X} - \mathbf{Y} = \begin{bmatrix} 4 & -7 \\ 8 & -1 \end{bmatrix}$$

Problem 7: Find $\mathbf{AB}$.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and}$$

$$\mathbf{B} = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$$

Solution 7:

Step 1: Check dimensions. $\mathbf{A}$ is 2×2 , $\mathbf{B}$ is 2×2 . Product is 2×2 . [OK]

Step 2: Row 1 of $\mathbf{A} \cdot$ Col 1 of $\mathbf{B}$: $(1)(-1) + (2)(2) = 3$

Step 3: Row 1 of $\mathbf{A} \cdot$ Col 2 of $\mathbf{B}$: $(1)(0) + (2)(3) = 6$

Step 4: Row 2 of $\mathbf{A} \cdot$ Col 1 of $\mathbf{B}$: $(3)(-1) + (4)(2) = 5$

Step 5: Row 2 of $\mathbf{A} \cdot$ Col 2 of $\mathbf{B}$: $(3)(0) + (4)(3) = 12$

$$\mathbf{AB} = \begin{bmatrix} 3 & 6 \\ 5 & 12 \end{bmatrix}$$

Problem 8: Find $\mathbf{CD}$.

Quiz 2: Determinants, Inverses & Systems

1. Find the determinant of $\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$.
2. Find the inverse of $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$.
3. Solve using Cramer's Rule: $x - 2y = 3$, $3x + y = 2$.
4. Write the augmented matrix for: $x + z = 5$, $2x + y = 0$, $y - z = 1$.

Comprehensive Unit Test

Part A: Multiple Choice

1. If matrix A is 3×2 and matrix B is 2×4 , what are the dimensions of AB?
a) 3×4 b) 2×2 c) 4×3 d) Undefined
2. If $\det(A) = 0$, then matrix A is:
a) Invertible b) Singular c) Identity d) Symmetric

Part B: Short Answer

3. Find $3A - 2B$, where $A = [-1, 4 ; 0, 2]$ and $B = [2, 0 ; -3, 1]$.
4. Calculate the determinant of $M = [2, 1, -1 ; 0, 3, 4 ; -2, 5, 6]$.
5. Find the inverse of $P = [7, 3 ; 2, 1]$. Verify that $PP^{-1} = I$.

Part C: Extended Response

6. Solve the following system completely using Gaussian Elimination (RREF). Label all row operations.

$$x + 2y - z = 3$$

$$2x + 5y + z = 8$$

$$-x + y + 3z = -1$$

7. A company produces Product X and Product Y. Product X requires 2 hours of machining and 1 hour of finishing. Product Y requires 1 hour of machining and 3 hours of finishing. Machining costs \$15/hr, and finishing costs \$20/hr. Use matrix multiplication to find the total cost to produce one unit of Product X and one unit of Product Y.

MATRICES & LINEAR SYSTEMS

COMPLETE UNIT | GRADES 10 - 12

90-PAGE FULL PDF

Guided notes, worked examples, practice, assessments, and full solutions in one complete unit.

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- Zero and identity matrices

TOPIC 2 | MATRIX MULTIPLICATION

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- 2×2 and 3×3 row-by-column products
- Non-commutativity, powers, transformation composition

TOPIC 3 | DETERMINANTS & INVERSES

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- 2×2 inverse formula
- 3×3 inverse by row reduction; verification

TOPIC 4 | SOLVING LINEAR SYSTEMS

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- Cramer's Rule for 2×2 and 3×3 systems
- Gaussian elimination, RREF, system classification

TOPIC 5 | REAL-WORLD APPLICATIONS

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- Business inventory and cost analysis
- Leontief input-output models and transformations

PRACTICE SETS

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- 20** Determinant Problems
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- Key Takeaways and key formulas
- Quick-reference summaries
- Common Mistakes callouts
- Guided practice and worked examples

FULL RESOURCE CONTENTS AT A GLANCE

Visual guided notes, worked examples, five topic areas, five practice sets, a cumulative review, two quizzes, a comprehensive unit test, and full solutions.