

★ TPT PREVIEW ★

GRADE 8 · COMPLETE MATH UNIT

Linear Functions *Modeling*

Rate of Change · Initial Value · Real-World Context

91 PAGES

160+ PROBLEMS

FULL SOLUTIONS

PREVIEW EDITION · A curated 10-page sample of the complete 91-page unit

UM PRINTABLES

PREMIUM TEACHING RESOURCES

CCSS 8.F.A.1–3 · 8.F.B.4–5

UNIT INTRODUCTION

Unit Overview & CCSS Standards Map

CCSS 8.F.A.1–3 · 8.F.B.4–5

About This Unit

This complete unit takes Grade 8 students from their first encounter with linear functions as formal mathematical objects through confident real-world modeling and qualitative graph analysis. The four topic progressions are designed to be taught sequentially, each building directly on the skills developed in the previous one. By the end of the unit, students will be able to construct a linear function from a table, graph, two coordinate pairs, or a verbal description; interpret the meaning of slope and y-intercept in a real-world context; use linear models to predict values and identify when a quantity reaches a target; and describe and sketch graphs qualitatively.

The unit is engineered to be highly usable for both teachers and students. Every worked example displays the full slope calculation with Δy and Δx values labelled explicitly, full substitution to find the y-intercept, the final equation written and labelled, and a complete contextual interpretation of both m and b . Student-facing guided notes include fill-in-the-blank modeling templates and interpretation sentence frames so that students can practice the academic language of linear functions while they practice the math. Every problem set is paired with full worked solutions, not just final answers — students can follow the reasoning on every problem.

Learning Targets

- **Construct** a linear function in $y = mx + b$ form from a table, graph, two (x, y) pairs, or a verbal description.
- **Interpret** the rate of change (m) and initial value (b) in context using complete sentences.
- **Distinguish** meaningful from meaningless extrapolation based on the contextual domain.
- **Model** real-world situations with linear functions, predict values, and find when a target quantity is reached.
- **Reason** about the contextual domain of a linear model.
- **Describe** graphs qualitatively using increasing / decreasing / constant and linear / nonlinear vocabulary.
- **Sketch** a graph from a verbal description, and match graphs to descriptions.

TOPIC 1 OF 4 • BUILD THE FUNCTION

Topic 1 — Constructing a Function from Data

CCSS 8.F.A.1 • 8.F.B.4

Big Idea

A **linear function** has the form $y = mx + b$, where **m** is the *slope* (rate of change) and **b** is the *y-intercept* (initial value when $x = 0$).

No matter what form the data comes in — a table, a graph, two coordinate pairs, or a verbal description — your job is always the same: **(1) find m, (2) find b, (3) write $y = mx + b$.**

Modeling Templates — Fill These In

Use these templates every time you construct a linear function. Photocopy this page, glue it into your math notebook, or keep it as a reference card. The templates work for every representation (table, graph, two points, verbal).

Template A — Find the slope m

$$m = \Delta y / \Delta x = (y_2 - y_1) / (x_2 - x_1)$$

Pick two points: $(x_1, y_1) = (_, _)$ and $(x_2, y_2) = (_, _)$

$$m = (_ - _) / (_ - _) = _ / _ = _$$

Template B — Find the y-intercept b

Method 1 (if a point with $x = 0$ is given): read b directly from the table / graph.

Method 2 (use any point (x, y) and your value of m):

$$\text{Substitute: } y = m \cdot x + b \rightarrow _ = (_)(_) + b$$

$$\text{Solve for b: } b = _$$

Template C — Write the final equation

$$y = _ x + _$$

Always check by substituting a known point that was NOT used to find m or b.

$$\text{Check: } y = (_)(_) + _ = _ \text{ (should equal known y-value)}$$

Interpretation Sentence Frames

Once you have m and b, use these frames to write your interpretation in complete sentences. Fill in the blanks based on the context.

For slope (m):

"For every increase of 1 in _____ (the input), the _____ (the output) changes by _____ units."

WORKED EXAMPLES

Topic 1 — Worked Examples (Four Representations)

CCSS 8.F.A.1 · 8.F.B.4

Below are four worked examples — one for each way you might receive linear data. Each shows the full slope calculation, substitution for b , final equation, and contextual interpretation. Use these as models for your own work.

Example 1 — From a Table

A linear function is shown in the table below. Write the equation in $y = mx + b$ form.

x : 0 1 2 3 4

y : 5 8 11 14 17

Step 1 — Find the slope $m = \Delta y / \Delta x$

Pick two points: $(0, 5)$ and $(1, 8)$

$$\Delta y = 8 - 5 = 3$$

$$\Delta x = 1 - 0 = 1$$

$$m = \Delta y / \Delta x = 3 / 1 = 3$$

Step 2 — Substitute a point to find b

Use point $(0, 5)$ — this is the y -intercept!

Substitute into $y = mx + b$: $5 = (3)(0) + b$

$$5 = 0 + b \rightarrow b = 5$$

Step 3 — Write the final equation

$$y = 3x + 5$$

Step 4 — Interpret m and b in context

$m = 3$: For every 1-unit increase in x , y increases by 3.

$b = 5$: When $x = 0$, $y = 5$ (this is the initial value).

Interpretation Worked Example

A pizza restaurant charges \$11 per pizza plus a \$3 delivery fee. The total cost C for p pizzas is modeled by $C = 11p + 3$.

Step 1 — Find the slope $m = \Delta y / \Delta x$

Rate: \$11 per pizza $\rightarrow m = 11$

Initial: \$3 delivery fee $\rightarrow b = 3$

(The slope and y-intercept were identified from the verbal description.)

Step 2 — Substitute a point to find b

b is given directly: $b = 3$ (the \$3 delivery fee).

Verify: at $p = 0$, $C = 11(0) + 3 = 3$ ◻

Step 3 — Write the final equation

$$C = 11p + 3$$

Step 4 — Interpret m and b in context

Interpretation of m (slope = 11): For every additional pizza ordered, the total cost increases by \$11.

Interpretation of b (y-intercept = 3): When 0 pizzas are ordered, the total cost is \$3. This \$3 is the delivery fee charged regardless of how many pizzas are purchased — even if you order nothing.

Meaningful vs. Meaningless Extrapolation

A linear model is only valid within a certain range of input values. Using the model outside that range is called **extrapolation**. Sometimes extrapolation is meaningful, and sometimes it is meaningless — you have to think about the real-world context.

Meaningful Extrapolation	Meaningless Extrapolation
Using a cost function $C = 1.50m + 8$ to predict the cost of renting 25 movies in a month — the model still applies, the answer is reasonable.	Using a candle height function $h = -0.4t + 24$ to predict the height after 100 hours — the candle is gone by hour 60, so the prediction ($h = -16$) is impossible.
Using a savings function $B = 5w + 200$ to predict the balance after 52 weeks — savings grow steadily, the answer is reasonable.	Using a population function $P = -8y + 1200$ to predict population after 200 years — the population reaches 0 at year 150, so the prediction ($P = -400$) is impossible.

How to Identify the Domain

The **domain** of a function is the set of all valid input values. For real-world linear functions, the domain is almost always restricted by physical or practical limits. Ask yourself these questions:

- Can the input be negative? (e.g., you cannot have negative miles, negative time, negative texts)
- Is there a physical maximum? (e.g., a tank holds a fixed volume — you cannot add more water once it is full)
- Is there a target value where the model stops making sense? (e.g., once a candle burns out, height cannot go negative)
- Are there natural limits in the context? (e.g., a phone plan has a maximum number of texts in a billing cycle)

ANSWER KEY WITH MODEL SENTENCES

Topic 2 — Model Answer Sentences

CCSS 8.F.B.4

Below are model interpretive sentences for each problem. Your wording may differ slightly — that is fine. Focus on whether your sentences (a) name the variables in context, (b) include units, and (c) clearly distinguish the meaning of m from the meaning of b .

Problem 1.	<i>A car rental costs \$45 per day plus a one-time \$20 fee. The total cost is modeled by $C = 45d + 20$, where d = days rented.</i>
m:	For every additional day the car is rented, the total cost increases by \$45.
b:	When the car is rented for 0 days, the cost is \$20 (the one-time fee). This represents the base fee charged regardless of rental duration.
Domain:	$d \geq 0$ (you cannot rent a car for a negative number of days).
Extrap:	Meaningful for d up to a few weeks. Beyond 30+ days, long-term rental discounts likely apply, so the linear model may break down.
Problem 2.	<i>The temperature in degrees Celsius is modeled by $T = -0.6h + 15$, where h = altitude in kilometers.</i>
m:	For every 1 km increase in altitude, the temperature decreases by 0.6°C .
b:	At altitude $h = 0$ km (sea level), the temperature is 15°C .
Domain:	$0 \leq h \leq -12$ (within the troposphere).
Extrap:	Meaningful within the troposphere (~ 12 km). Beyond that, temperature behavior changes — the linear model is meaningless at very high altitudes.
Problem 3.	<i>A candle burns according to $h = -0.4t + 24$, where h = height in cm, t = time in hours.</i>
m:	For every additional hour, the candle's height decreases by 0.4 cm.
b:	At time $t = 0$ (before burning), the candle is 24 cm tall.
Domain:	$0 \leq t \leq 60$ (candle is gone after $h = 0$, i.e., $t = 60$ hours).
Extrap:	Meaningful for $0 \leq t \leq 60$. Beyond $t = 60$, the candle has burned completely; predicting negative heights is meaningless.
Problem 4.	<i>A plumber charges \$80 per hour plus a \$50 service call fee. Total cost: $C = 80h + 50$.</i>
m:	For every additional hour of work, the cost increases by \$80.
b:	When the plumber works 0 hours, the cost is \$50 — this is the service-call fee charged just for showing up.
Domain:	$h \geq 0$ (cannot have negative hours of work).
Extrap:	Meaningful for typical job lengths (1–10 hours). For very large h , a project-rate contract might replace the hourly model.

TOPIC 3 OF 4 · APPLY THE MODEL

Topic 3 — Modeling Real-World Situations

CCSS 8.F.B.4

Big Idea

Many real-world situations can be modeled by linear functions: costs that grow per unit, quantities that drain or fill at a constant rate, distances that grow with time at constant speed. Once you have a linear model, you can **predict** values and **solve** for when a target quantity is reached.

Identifying a Linear Relationship

How can you tell whether a real-world situation can be modeled by a linear function? Use the **constant-difference test**: if the output changes by the same amount for every equal increase in the input, the relationship is linear.

- **Cost-plus-rate**: "Flat fee plus \$X per unit" → linear.
- **Constant draining/filling**: "Loses/gains X per minute at a steady rate" → linear.
- **Constant speed**: "Travels at X mph" → linear distance over time.
- **Constant growth/decay**: "Loses X per year" or "gains X per week" → linear.

Watch out for **non-linear** situations: compound interest (grows faster over time), accelerating motion, exponential population growth, and any context where the rate itself changes.

The Four-Step Modeling Process

For every real-world modeling problem, follow these four steps. Memorize them; they will guide every problem in this topic.

1

Identify the rate of change (m) and initial value (b).

Read the problem carefully. Look for phrases like "starts at," "initially," "base fee," or "per unit" — these reveal b and m respectively. Watch the sign: draining, losing, or decreasing → negative slope; filling, gaining, or increasing → positive slope.

2

Write the function in $y = mx + b$ form.

Substitute the values of m and b into the template. Use a context-appropriate variable name (e.g., C for cost, V for volume, h for height, d for distance).

3

Predict by substituting into the function.

To predict y at a given x, substitute the x value into the equation and evaluate. Always include units in your final answer.

4

Solve for when a target is reached.

To find when y equals a target value, set y equal to that target and solve for x. Check the answer against the meaningful domain.

Worked Example — Full Modeling Process

PRACTICE SET

Topic 4 — 15-Problem Qualitative Graph Analysis Set

CCSS 8.F.B.5

This set has three parts: **Problems 1–6** ask you to describe given graphs qualitatively using the three-part protocol. **Problems 7–11** ask you to sketch a graph from a verbal description — use the blank grids provided. **Problems 12–15** ask you to match a description to the correct graph. Full descriptions and reference sketches follow.

Problems 1–6: Describe Given Graphs

1. The graph shows a straight line rising from the lower-left to the upper-right of the coordinate plane, passing through $(0, 1)$ and $(3, 7)$. Describe the graph qualitatively.

2. The graph shows a curve that starts low on the left, rises steeply, then flattens out on the right. Describe the graph qualitatively.

3. The graph shows a straight horizontal line at $y = 4$. Describe the graph qualitatively.

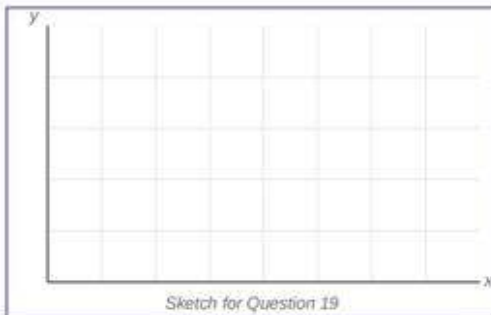
4. The graph shows a straight line decreasing from upper-left to lower-right, passing through $(0, 6)$ and $(4, 0)$. Describe the graph qualitatively.

5. The graph shows a curve shaped like a parabola opening downward, peaking at $(3, 9)$. Describe the graph qualitatively.

6. The graph shows a step pattern: flat at $y = 1$ from $x = 0$ to $x = 2$, then flat at $y = 3$ from $x = 2$ to $x = 4$, then flat at $y = 5$ from $x = 4$ to $x = 6$. Describe the graph qualitatively.

18. A graph is a parabola opening upward. Describe it qualitatively (linear/nonlinear, increasing/decreasing, any min/max).

19. Sketch a graph of a linear decreasing function with y-intercept at $(0, 8)$ and x-intercept at $(4, 0)$.



20. Match: "linear, constant function at $y = 3$ ". Choose from: A) Increasing line, B) Decreasing line, C) Horizontal line at $y = 3$, D) Parabola.

Solution 20. Match: "linear, constant function at $y = 3$ ". Choose from: A) Increasing line, B) Decreasing line, C) Horizontal line at $y = 3$, D) Parabola.

Steps: A horizontal line at $y = 3$ is constant (slope = 0).
Answer: C

Answer: C

What the Full Unit Contains

GRADE 8 • LINEAR FUNCTIONS MODELING

79

PAGES

160

PROBLEM ITEMS

4

TOPICS

5

CCSS STANDARDS

FOUR TOPIC PROGRESSIONS

- Topic 1 — Constructing a Function from Data: guided notes, worked examples, and a 35-problem function construction drill with solutions.
- Topic 2 — Interpreting Rate of Change & Initial Value: sentence frames, a 30-problem contextual interpretation activity, and model answer sentences.
- Topic 3 — Modeling Real-World Situations: guided notes, worked examples, a 25-problem modeling/prediction set, and solutions with interpretation.
- Topic 4 — Qualitative Analysis of Graphs: guided notes, worked examples, a 15-problem graph analysis set, and full descriptions/sketch guidance.

ACROSS THE FULL PDF

- Unit Overview & CCSS Standards Map covering 8.F.A.1, 8.F.A.2, 8.F.A.3, 8.F.B.4, and 8.F.B.5.
- Cumulative Unit Review with 25 mixed problems across all four topics.
- Unit Quiz with 10 problems and an answer key.
- Comprehensive Unit Test with 20 problems, followed by complete worked solutions for all 20 test problems.
- Student-facing guided notes include fill-in-the-blank modeling templates and interpretation sentence frames.

CONTENT AT A GLANCE

105 topic practice + 25 review + 10 quiz + 20 unit-test items = 160 problem items

The full resource is organized to move from constructing linear functions, to interpreting rate of change and initial value, to real-world modeling and prediction, and finally to qualitative graph analysis. A pacing guide in the unit suggests approximately 12–15 class periods of 45 minutes, with flexibility based on student readiness.

[UM PRINTABLES.COM](https://www.umprintables.com)**Linear Functions Modeling**

Grade 8 • Complete 79-page unit • CCSS 8.F.A.1–3 • 8.F.B.4–5