

UM PRINTABLES • GRADE 8 MATHEMATICS

Dilations & Similarity Transformations

Complete Teaching Unit — Lessons, Drills, Activities, Assessments & Full Worked Solutions

CCSS 8.G.A.3

CCSS 8.G.A.4

CCSS 8.G.A.5

WHAT'S INSIDE

- | | |
|--|---|
| • 4 topic-progressions | • Guided visual notes |
| • 35-problem dilation drill | • 30-problem similarity & proportion set |
| • 20-problem indirect measurement activity | • 15-problem AA similarity application set |
| • Cumulative unit review | • Unit quiz + comprehensive test |
| • Full worked solutions for every drill, activity, review & assessment | • Teacher notes, vocab cards, warm-ups & exit tickets |

Designed for Teachers Pay Teachers (TPT) — premium quality bar.

Table of Contents

A complete Grade 8 unit on dilations and similarity transformations, aligned to CCSS 8.G.A.3, 8.G.A.4 and 8.G.A.5. Every section below is self-contained and may be printed and distributed as a standalone handout. This **preview** shows selected pages from the full unit.

Unit Overview & Pacing Guide	5
Vocabulary Cards	7
Topic 1: Dilations on the Coordinate Plane	9
1.0 Warm-Up · 1.1 Defining a Dilation · 1.2 Coordinate Rule (Origin)	
1.3 Non-Origin Centre · 1.4 Enlargements & Reductions · 1.5 Worked Examples	
1.6 Exit Ticket · 1.7 Teacher Notes	
Topic 2: Similarity via Transformations	16
2.0–2.6: Formal definition, similarity statements, sequences, examples	
Topic 3: Proportional Reasoning with Similar Figures	21
3.0–3.7: Proportions, perimeter/area ratios, indirect measurement	
Topic 4: AA Similarity Criterion & Multi-Step Problems	28
4.0–4.6: AA criterion, parallel lines, Pythagorean integration	
35-Problem Dilation Drill on the Coordinate Plane	34
30-Problem Similarity & Proportion Set	38
20-Problem Scale Factor & Indirect Measurement Activity	41
15-Problem AA Similarity Application Set	44
Cumulative Unit Review	47
Unit Quiz (Topics 1–2)	49
Comprehensive Unit Test	51
Answer Key & Full Worked Solutions	55

Unit Overview & Pacing Guide

This unit develops the concept of **similarity** from a transformations-based perspective, the central approach of the Grade 8 Common Core geometry strand. Students begin by performing single dilations on the coordinate plane, then combine dilations with rigid motions to define similarity formally. They then use proportional reasoning to find missing measurements in similar figures — including indirect measurement applications such as the shadow and mirror methods. The unit culminates with the AA similarity criterion and multi-step problems that integrate similarity with the Pythagorean Theorem and angle relationships.

The pacing below assumes a 55-minute period. Teachers on block schedules may combine Days 1-2, 3-4, etc. Differentiation tips appear in the teacher notes section of each topic.

Common Core State Standards (CCSS)

Standard	Focus	Where in this Unit
8.G.A.3	Describe the effect of dilations (and rigid motions) on coordinates.	Topic 1: Dilations — Lessons 1.1–1.4; Dilation Drill
8.G.A.4	Define similarity via a sequence of rigid motions + dilation; describe the sequence; use proportions.	Topic 2 & Topic 3; Similarity & Proportion Set; Indirect Measurement Activity
8.G.A.5	Establish angle facts, including the AA criterion for triangle similarity.	Topic 4: AA Similarity; AA Application Set; Unit Test (Part III)

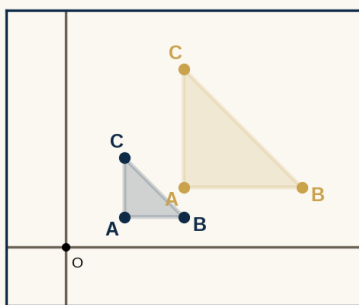
The full unit includes a complete two-week pacing guide, essential questions, and learning targets on the following page.

Vocabulary Cards

Twelve key terms anchor this unit. Each card below pairs a student-friendly definition with a visual cue. The first four cards are shown here; the full unit includes all twelve.

Dilation

A transformation that changes the size of a figure by a scale factor k relative to a fixed point called the *centre of dilation*. The image is the same shape as the pre-image, but not necessarily the same size.



Centre of Dilation

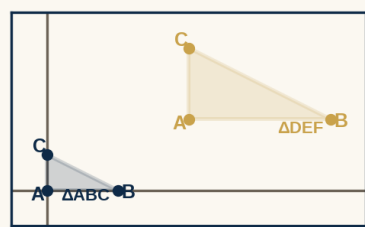
The fixed point that the figure grows toward or shrinks away from. Lines drawn from the centre through each pre-image vertex pass through the corresponding image vertex.

Scale Factor (k)

The ratio of any side length of the image to the corresponding side length of the pre-image. If $k > 1$ the dilation is an *enlargement*; if $0 < k < 1$ it is a *reduction*; if $k = 1$ the figure is unchanged.

Similar Figures

Two figures are **similar** if one can be obtained from the other by a sequence of rigid motions (translations, reflections, rotations) **combined with** a dilation. Corresponding angles are congruent and corresponding side lengths are proportional.



Topic 1: Dilations on the Coordinate Plane

1.0 Warm-Up: What Changes, What Stays the Same?

Real-World Hook — Photo Enlargement

A photographer has a 4 in $\times$ 6 in print. She uses software to enlarge it to 8 in $\times$ 12 in for a gallery frame. The image is the same shape, but bigger. What single number tells her how much bigger? That number is the **scale factor**, and the operation that produced the enlargement is a **dilation**.

Solve these in 5 minutes. Compare with a partner.

W1. Multiply each coordinate by 2. What happens to (3, 4), (−1, 5), and (0, −2)? What pattern do you notice about how far each point moves from the origin?

W2. A triangle has vertices A(1,1), B(3,1), C(1,2). Predict — without computing — whether multiplying every coordinate by $\frac{1}{2}$ will make the triangle larger, smaller, or the same. Explain your reasoning.

W3. Look at the photo-scale figure below. By what factor was the photo enlarged? How do you know?

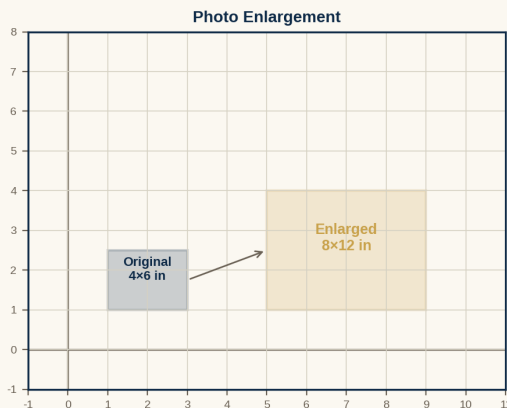


Figure: Figure: Original 4 $\times$ 6 in print and its enlargement.

1.1 Guided Notes — Defining a Dilation

A **dilation** is a transformation that produces an image that is the *same shape* as the pre-image, but *not necessarily the same size*. Every dilation is defined by two pieces of information:

- **Centre of dilation (C):** a fixed point. The image lies on rays drawn from the centre through each pre-image vertex.
- **Scale factor (k):** a positive number that tells how far each image vertex is from the centre, relative to its pre-image vertex. Specifically, $CP' = k \cdot CP$ for every point P with image P'.

Example 2 — Reduction about the Origin ($k = \frac{1}{2}$)

Rectangle PQRS has vertices $P(2, 2)$, $Q(6, 2)$, $R(6, 4)$, $S(2, 4)$. Dilate about the origin with $k = \frac{1}{2}$. State the image coordinates and classify.

Step 1 — Identify the rule and classify.

Because the centre is the origin and $k = \frac{1}{2}$, the rule is $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$. Each coordinate is multiplied by $\frac{1}{2}$ (i.e. divided by 2). Because $0 < \frac{1}{2} < 1$, we already know this dilation is a *reduction* — the image will be smaller than the pre-image and will lie between the origin and the pre-image.

Step 2 — Apply the rule vertex by vertex.

Multiply each coordinate by $\frac{1}{2}$. The substitutions and arithmetic are shown in the middle column; the final image coordinates are on the right.

Pre-image	Rule $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$	Image
P (2, 2)	$(\frac{1}{2} \cdot 2, \frac{1}{2} \cdot 2) = (1, 1)$	P' (1, 1)
Q (6, 2)	$(\frac{1}{2} \cdot 6, \frac{1}{2} \cdot 2) = (3, 1)$	Q' (3, 1)
R (6, 4)	$(\frac{1}{2} \cdot 6, \frac{1}{2} \cdot 4) = (3, 2)$	R' (3, 2)
S (2, 4)	$(\frac{1}{2} \cdot 2, \frac{1}{2} \cdot 4) = (1, 2)$	S' (1, 2)

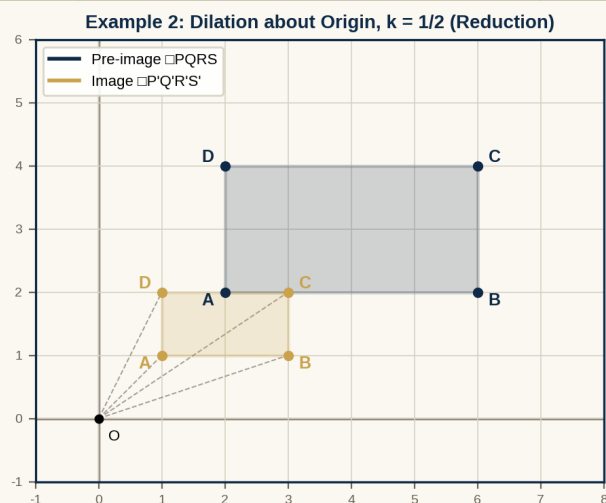


Figure: Rectangle PQRS (4×2) reduced to $P'Q'R'S'$ (2×1) about the origin with $k = \frac{1}{2}$. Each image coordinate is half its pre-image value.

Topic 3: Proportional Reasoning with Similar Figures

3.0 Warm-Up: The Power of Proportions

Real-World Hook — The Flagpole Shadow

How tall is the school flagpole? You can't exactly lean a ladder against it with a tape measure. But on a sunny day, you can stand next to it, measure your shadow and the flagpole's shadow, and use **similar triangles** to compute the height. This technique — **indirect measurement** — has been used for over 2,500 years, since the Greek mathematician Thales estimated the height of the Great Pyramid.

W1. Solve for x : $\frac{3}{4} = \frac{9}{x}$. Show your cross-multiplication.

W2. If a 6-ft-tall person casts a 4-ft shadow, what is the ratio of person-height to shadow-length? Express as a fraction in lowest terms.

W3. Two similar rectangles have widths 3 cm and 9 cm. What is the scale factor k ? If the smaller rectangle is 5 cm long, how long is the larger one?

3.1 Setting Up a Proportion

When two figures are similar, every pair of corresponding side lengths forms the same ratio k . This lets us set up a **proportion** — an equation stating that two ratios are equal — and solve for any unknown side.

The key is **matching corresponding sides**. A foolproof setup is:

$$\frac{(\text{preimage side A})}{(\text{preimage side B})} = \frac{(\text{image side A}')}{(\text{image side B'})}$$

Equivalently, you can use:

$$\frac{(\text{image side A}')}{(\text{preimage side A})} = \frac{(\text{image side B}')}{(\text{preimage side B})} = k$$

Either setup works as long as you are consistent. The most common student error is mixing directions (image/preimage on one side, preimage/image on the other), which produces the wrong answer. Always check that the units match on the top and bottom of each ratio.

35-Problem Dilation Drill on the Coordinate Plane

Work all 35 problems on a separate sheet or in your notebook. For each problem: (1) write the dilation rule, (2) apply it vertex-by-vertex, (3) state the image coordinates, (4) classify as enlargement, reduction, or identity. Mini-grids are provided for sketching on selected problems. Full worked solutions appear in the Answer Key section at the back of this unit.

Set A — Dilations about the Origin (Problems 1–15)

Apply the rule $(x, y) \rightarrow (kx, ky)$ to each pre-image vertex. State the image coordinates and classify the dilation.

1. Dilate $A(1, 1)$, $B(3, 1)$, $C(1, 2)$ by $k = 2$.

2. Dilate $A(2, 4)$, $B(6, 4)$, $C(6, 8)$, $D(2, 8)$ by $k = 3$.

3. Dilate $A(1, -1)$, $B(3, -1)$, $C(2, -4)$ by $k = 2$.

4. Dilate $A(4, 2)$, $B(8, 2)$, $C(8, 6)$, $D(4, 6)$ by $k = \frac{1}{2}$.

5. Dilate $A(2, 3)$, $B(4, 3)$, $C(4, 5)$, $D(2, 5)$ by $k = \frac{1}{2}$.

6. Dilate $A(-2, 1)$, $B(2, 1)$, $C(0, 5)$ by $k = 2$.

7. Dilate $A(0, 2)$, $B(4, 2)$, $C(2, 5)$ by $k = 1.5$.

8. Dilate $A(1, 0)$, $B(3, 0)$, $C(3, 4)$, $D(1, 4)$ by $k = \frac{1}{4}$.

... continued in the full unit (problems 9–35, Sets B and C, plus full worked solutions in the Answer Key).

Comprehensive Unit Test

Time: 50 minutes | **Total:** 100 points. Show all work. Calculators permitted for arithmetic only. Solutions in the Answer Key.

Part I — Multiple Choice (10 questions, 3 pts each = 30 pts)

1. The image of $(-2, 5)$ after a dilation about the origin with $k = 3$ is:

- A. $(-6, 15)$
- B. $(6, -15)$
- C. $(-6, 5)$
- D. $(-2, 15)$

2. A dilation with $k = 1.5$ is:

- A. an enlargement
- B. a reduction
- C. the identity
- D. a reflection

3. Dilation about (a, b) with scale factor k uses the rule:

- A. $(x, y) \rightarrow (kx, ky)$
- B. $(x, y) \rightarrow (a + kx, b + ky)$
- C. $(x, y) \rightarrow (a + k(x - a), b + k(y - b))$
- D. $(x, y) \rightarrow (k(x - a), k(y - b))$

4. If $\triangle ABC \sim \triangle DEF$ and $AB = 4$, $DE = 6$, $BC = 6$, then $EF =$

- A. 4
- B. 8
- C. 9
- D. 12

5. Two similar figures have $k = 5$. The area ratio (image/preimage) is:

... continued in the full unit (questions 6–10, Part II short answer, Part III extended response, plus full worked solutions).

Answer Key & Full Worked Solutions

Complete worked solutions for every drill, activity, review, and assessment in this unit: the 35-problem dilation drill, the 30-problem similarity & proportion set, the 20-problem indirect-measurement activity, the 15-problem AA application set, the cumulative review, the quiz, and the comprehensive test. Every coordinate computation is shown vertex-by-vertex; every proportion is set up formally, cross-multiplied, and solved with full arithmetic. (Warm-ups and exit tickets are intentionally left for in-class discussion; their answers follow directly from the worked examples and guided notes in each topic.)

Notation: Prime notation (e.g. A') denotes the image of vertex A . C or C_0 denotes the centre of dilation. k is the scale factor.

A. Dilation Drill (35 Problems)

Set A — Dilations about the Origin (1–15)

1. $k = 2$ (enlargement). $A(1,1) \rightarrow (2,2)$; $B(3,1) \rightarrow (6,2)$; $C(1,2) \rightarrow (2,4)$. Image: $A'(2,2)$, $B'(6,2)$, $C'(2,4)$.
2. $k = 3$ (enlargement). $A(2,4) \rightarrow (6,12)$; $B(6,4) \rightarrow (18,12)$; $C(6,8) \rightarrow (18,24)$; $D(2,8) \rightarrow (6,24)$. Image: $A'(6,12)$, $B'(18,12)$, $C'(18,24)$, $D'(6,24)$.
3. $k = 2$ (enlargement). $A(1,-1) \rightarrow (2,-2)$; $B(3,-1) \rightarrow (6,-2)$; $C(2,-4) \rightarrow (4,-8)$. Image: $A'(2,-2)$, $B'(6,-2)$, $C'(4,-8)$.
4. $k = \frac{1}{2}$ (reduction). $A(4,2) \rightarrow (2,1)$; $B(8,2) \rightarrow (4,1)$; $C(8,6) \rightarrow (4,3)$; $D(4,6) \rightarrow (2,3)$. Image: $A'(2,1)$, $B'(4,1)$, $C'(4,3)$, $D'(2,3)$.
5. $k = \frac{1}{2}$ (reduction). $A(2,3) \rightarrow (1,1.5)$; $B(4,3) \rightarrow (2,1.5)$; $C(4,5) \rightarrow (2,2.5)$; $D(2,5) \rightarrow (1,2.5)$. Image: $A'(1,1.5)$, $B'(2,1.5)$, $C'(2,2.5)$, $D'(1,2.5)$.

... continued in the full unit (solutions 6–35, Sets B and C, plus the Similarity, Indirect Measurement, AA, Review, Quiz, and Test solutions).

YOU'VE SEEN THE PREVIEW

WHAT IS INCLUDED IN THE COMPLETE 74-PAGE UNIT?

74

TOTAL PAGES

4

TOPICS

11

WORKED EXAMPLES

155

TOTAL PROBLEMS

TOPIC 1 • DILATIONS

Guided notes on dilation, centre and scale factor; origin and non-origin coordinate rules; enlargements, reductions and finding k ; 3 worked examples; exit ticket; teacher notes.

TOPIC 2 • SIMILARITY

Formal definition of similarity; similarity statements and corresponding parts; transformation sequences; 2 worked examples; exit ticket; teacher notes.

TOPIC 3 • PROPORTIONAL REASONING

Proportions and cross-multiplication; perimeter and area ratios; shadow and mirror methods for indirect measurement; 3 worked examples; exit ticket; teacher notes.

TOPIC 4 • AA SIMILARITY & MULTI-STEP

AA similarity criterion; parallel lines and similar triangles; integration with the Pythagorean Theorem; 3 worked examples; exit ticket; teacher notes.

PRACTICE, ASSESSMENTS & TEACHER MATERIALS

4 problem sets • 100 problems

35-problem Dilation Drill

30-problem Similarity & Proportion Set

20-problem Scale Factor & Indirect Measurement Activity

15-problem AA Similarity Application Set

Additional unit materials

20-problem Cumulative Unit Review

10-question Unit Quiz (Topics 1–2) • 30 points

25-question Comprehensive Unit Test • 100 points

12 Vocabulary Cards • warm-ups • exit tickets • teacher notes

UNIT STRUCTURE

A complete two-week pacing guide, essential questions and learning targets support the four-topic sequence. The unit is aligned to CCSS 8.G.A.3, 8.G.A.4 and 8.G.A.5.

UMPrintables.com

Preview information based on the complete resource contents.

SOLUTIONS

Full worked solutions are provided for every problem in the drill, indirect-measurement activity, AA/application set, cumulative review, unit quiz and comprehensive test.