

Congruence via Rigid Transformations

A premium, classroom-ready unit built around the five CCSS topic progressions for Grade 8 geometry.

From the coordinate rules of translations, reflections, and rotations to the informal proof of SSS, SAS, and ASA through sequences of rigid motions — this unit gives students visual guided notes, fully worked examples, and 100+ practice problems with complete solutions.

5 Topic Progressions

Guided Notes

100+ Problems

Full Solutions

Quiz & Unit Test

CCSS-Aligned

STANDARDS

8.G.A.1 – 8.G.A.3

AUDIENCE

Grade 8 Math

FORMAT

Print & Digital

UM PRINTABLES • GRADE 8 MATH

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CCSS 8.G.A.1 – 8.G.A.3

How to Use This Resource

8.G.A.1 – 8.G.A.3

Welcome to the **UM Printables Grade 8 Complete Unit on Congruence via Rigid Transformations**. This resource is built around the five-topic progression that underpins the Common Core State Standards for Grade 8 Geometry, clusters 8.G.A.1 through 8.G.A.3. The unit moves students from the concrete coordinate rules for translations, reflections, and rotations, through the definition of congruence as a sequence of rigid motions, and finally to an informal understanding of why the SSS, SAS, and ASA triangle congruence criteria actually work. Every section is designed so that a teacher can present the material directly from these pages and a student can use the same pages as a complete, self-contained study guide.

The structure is intentionally linear. Each topic opens with a short orientation and a list of learning targets, then introduces the coordinate rules with at least one fully worked example. Worked examples always list every vertex before and after the transformation, apply the coordinate rule explicitly to each vertex, show the complete ordered sequence as a mapping, and verify the congruence claim by checking preserved measurements (and angle measures when explicitly needed). After the worked examples, students encounter a set of practice problems whose full solutions appear later in the unit. The unit closes with a cumulative review, a unit quiz, and a comprehensive unit test, each with complete worked solutions so that grading and self-study are equally easy.

The Five Topic Progressions

Topic	Title	Focus
Topic 1	Properties of Rigid Transformations	Translations, reflections, and rotations — and the three properties they preserve: segment lengths, angle measures, and parallel lines.
Topic 2	Performing Transformations	Applying each coordinate rule vertex-by-vertex, building coordinate tables, and plotting both preimage and image on the same grid.
Topic 3	Congruence via Sequences	Defining congruence, writing transformation sequences, verifying by showing all corresponding sides and angles equal, writing congruence statements.
Topic 4	Describing Sequences	Given two congruent figures, systematically identifying a transformation sequence that maps one to the other.
Topic 5	Connecting to Triangle Congruence Criteria	Informal understanding of why SSS, SAS, and ASA work — by showing that the criteria force a unique rigid-transformation image.

Interactive Features for Students and Teachers

This unit adopts six researched interactive features that have been shown to improve student engagement and retention in middle-school mathematics. They appear as colored callout boxes throughout the unit so that students and teachers can spot them at a glance.

Feature	Purpose
Guided Notes	Yellow-tinted boxes with fill-in-the-blank lines that students complete as they read. Research on guided notes (Heward, 1994; Hamilton et al., 2000) shows they increase on-task behavior and accuracy of recorded content.

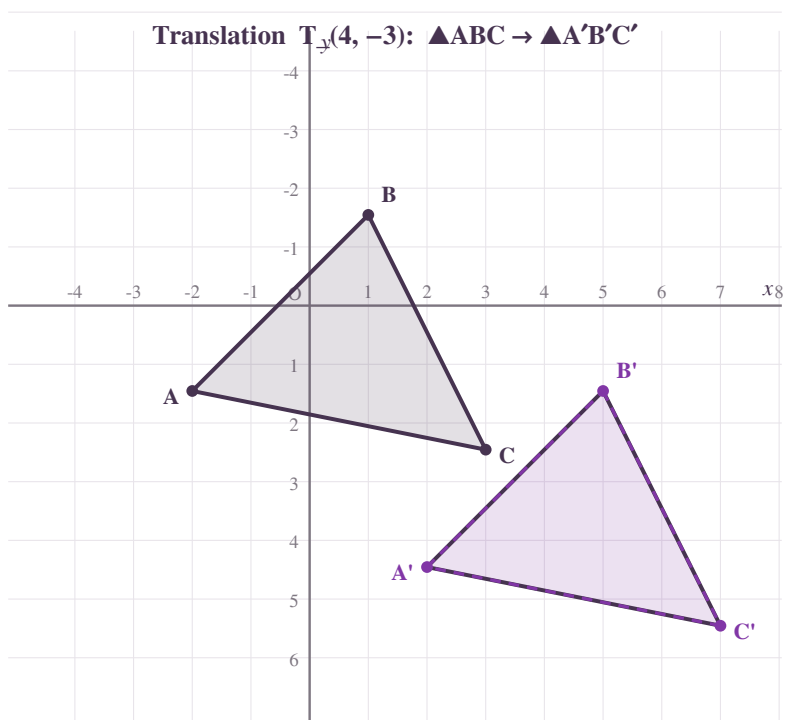


Figure 1.1 — Preimage $\triangle ABC$ (solid indigo) and image $\triangle A'B'C'$ (dashed purple) under $T_{(4, -3)}$.

Verification — Side Lengths Preserved

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

Side	Preimage length	Side	Image length	Equal?
AB	$\sqrt{((1 - (-2))^2 + (4 - 1)^2)} = \sqrt{(9 + 9)} = \sqrt{18} \approx 4.24$	A'B'	$\sqrt{((5 - 2)^2 + (1 - (-2))^2)} = \sqrt{(9 + 9)} = \sqrt{18} \approx 4.24$	✓
BC	$\sqrt{((3 - 1)^2 + (0 - 4)^2)} = \sqrt{(4 + 16)} = \sqrt{20} \approx 4.47$	B'C'	$\sqrt{((7 - 5)^2 + (-3 - 1)^2)} = \sqrt{(4 + 16)} = \sqrt{20} \approx 4.47$	✓
CA	$\sqrt{((-2 - 3)^2 + (1 - 0)^2)} = \sqrt{(25 + 1)} = \sqrt{26} \approx 5.10$	C'A'	$\sqrt{((2 - 7)^2 + (-2 - (-3))^2)} = \sqrt{(25 + 1)} = \sqrt{26} \approx 5.10$	✓

All three pairs of corresponding sides have equal lengths. Therefore $\triangle ABC \cong \triangle A'B'C'$, which confirms that $T_{(4, -3)}$ is a rigid transformation and preserves segment lengths (CCSS 8.G.A.1.a).

⚠ Watch Out!

A common error is to apply the translation to only one vertex and assume the others "move along". A translation moves **every** point of the figure by the same vector (a, b) . Apply $(x, y) \rightarrow (x + a, y + b)$ to **each** vertex independently.

✓ Your Turn

Triangle DEF has vertices D(0, 0), E(2, 3), F(5, 1). Apply the translation $T_{(-3, 5)}$. List the image vertices D', E', F'. (Answer appears in the solution to Practice Set A, problem A1.)

★ Teacher Note

Have students physically fold graph paper along the proposed line of reflection and check that the preimage lands exactly on the image. This kinesthetic check prevents the most common error: confusing reflection across $y = x$ with a 90° rotation about the origin.

1.5 Transformation Rule Reference Table (Master)

The single-page reference below collects every coordinate rule introduced in Topic 1. Students should reproduce this table from memory by the end of the unit. A blank reproducible version appears at the end of Practice Set A.

Transformation	Coordinate rule	Quick memory hook
Translation $T_{(a, b)}$	$(x, y) \rightarrow (x + a, y + b)$	Slide by vector (a, b)
Reflection over x-axis	$(x, y) \rightarrow (x, -y)$	Flip vertically
Reflection over y-axis	$(x, y) \rightarrow (-x, y)$	Flip horizontally
Reflection over $y = x$	$(x, y) \rightarrow (y, x)$	Swap coordinates
Reflection over $y = -x$	$(x, y) \rightarrow (-y, -x)$	Swap and negate both
Reflection over $x = k$	$(x, y) \rightarrow (2k - x, y)$	Vertical mirror
Reflection over $y = k$	$(x, y) \rightarrow (x, 2k - y)$	Horizontal mirror
Rotation 90° CCW	$(x, y) \rightarrow (-y, x)$	Quarter-turn left
Rotation 180°	$(x, y) \rightarrow (-x, -y)$	Half turn
Rotation 270° CCW	$(x, y) \rightarrow (y, -x)$	Quarter-turn right
Rotation 360°	$(x, y) \rightarrow (x, y)$	Identity

Table 1.5 — Master Transformation Rule Reference. Reproduce from memory by the end of the unit.

► Vocabulary

Preimage and Image — The **preimage** is the original figure before a transformation is applied. The **image** is the resulting figure after the transformation. We use prime notation: if A is a preimage vertex, A' (read "A prime") is its image.

► Vocabulary

Isometry — A synonym for **rigid transformation** — a transformation that preserves all distances. The prefix "iso-" means "same" and "metry" means "measure". Every translation, reflection, and rotation is an isometry; dilations (which scale distances) are not.

Side	Preimage length	Side	Image length	Equal?
GH	$\sqrt{((5-5)^2 + (1-4)^2)} = 3$	G'H'	$\sqrt{((1-4)^2 + (-5-(-5))^2)} = 3$	✓
HF	$\sqrt{((2-5)^2 + (4-1)^2)} = \sqrt{18}$	H'F'	$\sqrt{((4-1)^2 + (-2-(-5))^2)} = \sqrt{18}$	✓

All three pairs of corresponding sides are equal. Therefore $\triangle FGH \cong \triangle F'G'H'$.

⚠ Watch Out!

Notice that $\triangle FGH$ is a right triangle with the right angle at G. After the 270° rotation, the right angle is at G' — the **vertex correspondence** is preserved by the rotation. This is what we mean when we say angles are preserved: not just that the figure has the same angle measures, but that each specific angle at a specific vertex maps to the same angle at the image vertex.

Worked Example 2.3B — A Two-Step Sequence

Triangle ABC has A(0, 0), B(2, 0), C(1, 3). Apply the sequence **(1) translation $T_{(-1, 2)}$** , then **(2) reflection across the x-axis**. Build a coordinate table that shows the preimage, the intermediate image after Step 1, and the final image after Step 2. Plot all three figures on the same grid.

Vertex	Preimage	After $T_{(-1, 2)}$	Image label	After $r_{(x\text{-axis})}$
A	(0, 0)	(-1, 2)	A'	(-1, -2)
B	(2, 0)	(1, 2)	B'	(1, -2)
C	(1, 3)	(0, 5)	C'	(0, -5)

Ordered sequence as a mapping:

$$A(0, 0) \rightarrow (-1, 2) \rightarrow A'(-1, -2)$$

$$B(2, 0) \rightarrow (1, 2) \rightarrow B'(1, -2)$$

$$C(1, 3) \rightarrow (0, 5) \rightarrow C'(0, -5)$$

3.3 Verifying Congruence

To verify that two figures are congruent, you must show two things:

- (1) **All corresponding sides are equal.** Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ on every pair of corresponding vertices, and confirm each pair gives the same value.
- (2) **All corresponding angles are equal.** Use the slope formula to compute the slopes of the two sides forming each angle. The angle θ between two lines of slope m_1 and m_2 satisfies $\tan \theta = |(m_1 - m_2) / (1 + m_1 m_2)|$ (when $m_1 m_2 \neq -1$). Alternatively, use the dot-product formula $\cos \theta = (u \cdot v) / (|u| |v|)$ for the two vectors forming the angle.

Once both are confirmed, you can write the congruence statement with the correct vertex correspondence.

Worked Example 3.3A — Verifying Congruence

Figure 1 is triangle ABC with A(0, 0), B(4, 0), C(2, 3). Figure 2 is triangle DEF with D(-2, 1), E(-6, 1), F(-4, 4). Verify that $\triangle ABC \cong \triangle DEF$ by showing all three pairs of corresponding sides are equal and all three pairs of corresponding angles are equal. Then describe a transformation sequence that maps $\triangle ABC$ to $\triangle DEF$.

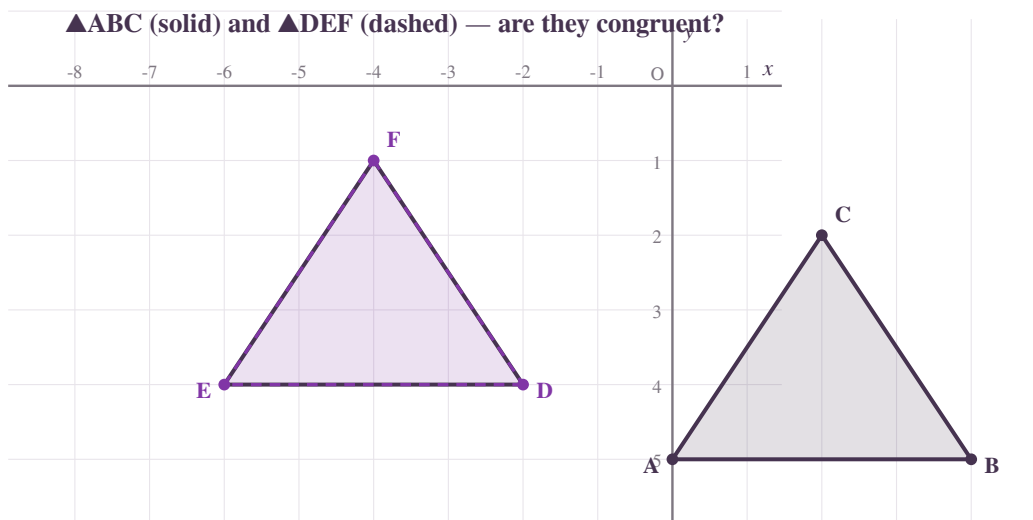


Figure 3.1 — Visual inspection suggests congruence; we now verify by computation.

Step 1 — Verify corresponding sides

Preimage side	Length	Image side	Length	Equal?
AB	$\sqrt{((4-0))^2 + (0-0)^2} = 4$	DE	$\sqrt{((-6-(-2)))^2 + (0-1)^2} = 4$	✓
BC	$\sqrt{((2-4))^2 + (3-0)^2} = \sqrt{13}$	EF	$\sqrt{((-4-(-6)))^2 + (4-1)^2} = \sqrt{13}$	✓
CA	$\sqrt{((0-2))^2 + (0-3)^2} = \sqrt{13}$	FD	$\sqrt{((-2-(-4)))^2 + (1-4)^2} = \sqrt{13}$	✓

CCSS 8.G.A.1, 8.G.A.3

Practice Set A — 40-Problem Transformation

Application Drill

8.G.A.1, 8.G.A.3

Apply the indicated transformation to each preimage. Record every image vertex in your notebook. Full solutions appear on the pages immediately following the problem set.

A.1 Translations (Problems A1–A10)

- A1.** Triangle DEF: D(0, 0), E(2, 3), F(5, 1). Apply $T_{(-3, 5)}$.
- A3.** Triangle PQR: P(1, 1), Q(4, 2), R(2, 5). Apply $T_{(0, -3)}$.
- A5.** Triangle ABC: A(-2, -1), B(1, -1), C(-1, 3). Apply $T_{(3, 4)}$.
- A7.** Triangle TUV: T(3, 0), U(6, 0), V(4, 4). Apply $T_{(-6, 2)}$.
- A9.** Hexagon ABCDEF: A(0, 0), B(2, 0), C(3, 2), D(2, 4), E(0, 4), F(-1, 2). Apply $T_{(2, -3)}$.
- A2.** Square ABCD: A(0, 0), B(3, 0), C(3, 3), D(0, 3). Apply $T_{(4, -2)}$.
- A4.** Pentagon VWXYZ: V(1, 0), W(3, 2), X(2, 4), Y(0, 3), Z(-1, 1). Apply $T_{(5, 5)}$.
- A6.** Quadrilateral KLMN: K(0, 0), L(2, 0), M(2, 2), N(0, 2). Apply $T_{(-5, -1)}$.
- A8.** Segment AB: A(-3, -2), B(2, 4). Apply $T_{(1, -1)}$.
- A10.** Triangle XYZ: X(-4, 2), Y(-1, 5), Z(1, 1). Apply $T_{(6, -6)}$.

A.2 Reflections (Problems A11–A22)

- A11.** Triangle TUV: T(-3, 0), U(0, 4), V(2, -1). Reflect over the y-axis.
- A13.** Quadrilateral PQRS: P(1, 2), Q(4, 1), R(5, 4), S(2, 5). Reflect over $y = x$.
- A15.** Pentagon ABCDE: A(0, 0), B(2, 1), C(3, 3), D(1, 4), E(-1, 2). Reflect over the x-axis.
- A17.** Square ABCD: A(0, 0), B(2, 0), C(2, 2), D(0, 2). Reflect over $y = 3$.
- A19.** Segment JK: J(-3, -2), K(2, 5). Reflect over the x-axis.
- A21.** Quadrilateral RSTU: R(1, 1), S(4, 1), T(4, 4), U(1, 4). Reflect over $y = x$.
- A12.** Triangle ABC: A(1, 2), B(4, 1), C(2, 5). Reflect over the x-axis.
- A14.** Triangle LMN: L(2, 0), M(5, 1), N(3, 4). Reflect over $y = -x$.
- A16.** Triangle DEF: D(-2, 1), E(1, 4), F(3, 0). Reflect over $x = 1$.
- A18.** Triangle GHI: G(1, 1), H(5, 1), I(3, 4). Reflect over $x = -1$.
- A20.** Triangle MNP: M(0, 0), N(4, 0), P(2, 3). Reflect over $y = -x$.
- A22.** Triangle VWX: V(-2, 3), W(0, 5), X(2, 3). Reflect over the y-axis.

A.3 Rotations about the Origin (Problems A23–A32)

- A23.** Triangle ABC: A(1, 1), B(4, 2), C(2, 5). Rotate 180° .
- A25.** Square ABCD: A(0, 0), B(3, 0), C(3, 3), D(0, 3). Rotate 270° CCW.
- A24.** Triangle DEF: D(2, 0), E(5, 1), F(3, 4). Rotate 90° CCW.
- A26.** Triangle GHI: G(-2, 1), H(1, 4), I(3, 0). Rotate 90° CCW.

Comprehensive Unit Test — Complete Worked Solutions

Part I — Multiple Choice

#	Answer and justification
T1	(b) A translation. The rule $(x, y) \rightarrow (x + 3, y - 4)$ adds a constant vector $(3, -4)$ to every point.
T2	(b) 90° CW. A 270° CCW rotation is the same as a 90° CW rotation (both use rule $(x, y) \rightarrow (y, -x)$).
T3	(c) $(-5, -3)$. The rule for reflection over $y = -x$ is $(x, y) \rightarrow (-y, -x)$. So $(3, 5) \rightarrow (-5, -3)$.
T4	(d) Distance from the origin is NOT preserved. A translation moves every point away from the origin. Rigid transformations preserve distances between pairs of points, not distance from a fixed point.
T5	(b) One can be obtained from the other by a sequence of rigid transformations. This is the CCSS definition of congruence.

Part II — Computation (full coordinate tables)

T6. $T_{(5, -1)}$ applied to $\triangle ABC$.

Vertex	Preimage	Apply $(x, y) \rightarrow (x + 5, y - 1)$	Image
A	(0, 0)	$(0+5, 0+(-1))$	$A'(5, -1)$
B	(4, 0)	$(4+5, 0+(-1))$	$B'(9, -1)$
C	(2, 3)	$(2+5, 3+(-1))$	$C'(7, 2)$

T7. $r_{y\text{-axis}}$ applied to $\triangle DEF$.

Vertex	Preimage	Apply $(x, y) \rightarrow (-x, y)$	Image
D	(1, 2)	$(-1, 2)$	$D'(-1, 2)$
E	(4, 2)	$(-4, 2)$	$E'(-4, 2)$
F	(2, 5)	$(-2, 5)$	$F'(-2, 5)$

T8. $R_{90^\circ \text{ CCW}}$ applied to $\triangle GHI$.

Vertex	Preimage	Apply $(x, y) \rightarrow (-y, x)$	Image
G	(2, 0)	$(0, 2)$	$G'(0, 2)$
H	(5, 0)	$(0, 5)$	$H'(0, 5)$
I	(3, 4)	$(-4, 3)$	$I'(-4, 3)$

T9. R_{180° applied to $\triangle JKL$.

Vertex	Preimage	Apply $(x, y) \rightarrow (-x, -y)$	Image
J	(0, 0)	$(0, 0)$	$J'(0, 0)$
K	(3, 0)	$(-3, 0)$	$K'(-3, 0)$

What's Inside the Full 68-Page Unit

Everything you need to teach CCSS 8.G.A.1 through 8.G.A.3 — in one resource.

68

PAGES

5

TOPICS

100+

PROBLEMS

4

PRACTICE SETS

3

ASSESSMENTS

5 Topic Progressions

Carefully sequenced for conceptual understanding

- 1 **Properties of Rigid Transformations**
Translations, reflections, rotations + 3 preserved properties
- 2 **Performing Transformations**
Vertex-by-vertex procedure, coordinate tables, plotting both figures
- 3 **Congruence via Sequences**
Defining congruence, writing sequences, verifying sides + angles
- 4 **Describing Sequences**
Systematic 5-step method to identify a transformation sequence
- 5 **Triangle Congruence Criteria**
Informal SSS, SAS, ASA proofs via rigid transformations

Practice & Assessment

Full worked solutions for every problem

- 40 **Transformation Drill**
Translations · Reflections · Rotations · Multi-step
- 30 **Congruence ID**
Identify · Describe sequence · Write statement
- 20 **Sequence Writing**
Write sequence · Verify vertex-by-vertex
- 10 **Properties Reasoning**
Justify claims · Reference CCSS sub-standards

Assessments Included

- **Cumulative Review**
25 mixed problems + concise answer key
- **Unit Quiz**
12 questions, 30 min, 34 pts + answer key
- **Comprehensive Unit Test**
20 questions, 60 min, 100 pts + FULL worked solutions

6 Researched Interactive Features

Guided Notes

Fill-in-the-blank

Watch Out!

Common error flags

Your Turn

Quick checkpoints

Real-World

Authentic contexts

Teacher Note

Pedagogical tips

Vocabulary

Key term boxes

CCSS Standards Covered

8.G.A.1 · 8.G.A.1.a · 8.G.A.1.b · 8.G.A.1.c · 8.G.A.2 · 8.G.A.3

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